1 Compound Interest Models

1.1 Compound Interest Formula with n Compounding Periods

Definition 1. For an initial principal P, annual interest rate r (expressed as a decimal), time t in years, and n compounding periods per year, the compound interest formula is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \tag{1}$$

where A is the final amount.

Example 1. If you invest \$5,000 at an annual interest rate of 4% compounded quarterly for 10 years, find the final amount and how long it takes to reach $\$9000$.

1.2 Continuously Compounded Interest

Definition 2. As the number of compounding periods approaches infinity, we obtain the continuously compounded interest formula:

$$A = Pe^{rt} \tag{2}$$

where $e \approx 2.71828$ is the base of the natural logarithm.

Example 2. If you invest \$10,000 at an annual interest rate of 5% compounded continuously for 8 years, find the final amount and how long it takes for the amount to double.

2 Exponential Growth and Decay Models

2.1 Exponential Growth Model

Definition 3. The quantity Q at time t with initial value Q_0 and growth rate k is given by:

$$Q = Q_0 e^{kt} \quad \text{(continuous)} \tag{3}$$

where $k > 0$ is the continuous growth rate.

Example 3. A bacterial colony starts with 500 bacteria and grows at a continuous rate of 15% per hour. a) How many bacteria will be present after 12 hours? b) How long it takes for the bacteria to reach 1500?

2.2 Exponential Decay Model

Definition 4. The quantity Q at time t with initial value Q_0 and decay rate k is given by:

$$Q = Q_0 e^{-kt} \quad \text{(continuous)} \tag{4}$$

where $k > 0$ is the continuous decay rate.

Example 4. A radioactive substance has an initial mass of 50 grams and decays at a continuous rate of 3% per day. a) What will be the mass after 30 days? b) How long until the mass is half of what it started?

Example 5. A population grows from 10,000 to 15,000 in 5 years. Find the continuous growth rate.