Math 140: Chapter 4 Set Theory

Sets

What is a set? A set is a collection of distinct objects called elements of the set. We usually use capital letters to denote sets A, B, C, ...

Example 1. Let A = The set of all whole numbers between 1 and 6.

- Word description: The set of all whole numbers between 1 and 6.
- Listing method: (roster) $A = \{1, 2, 3, 4, 5, 6\}$
- Set-builder notation: $A = \{x \mid$ condition to be considered an element of $A\}$

$A = \{x \mid 1 \leq x \leq 6$ and x is a whole number$\}$ (such that)

The set with no elements is called the empty set and it is denoted $\emptyset$ → notation for empty set or $\{\}$

To specify that an element belongs to a set we write

$1 \in A$ ↪ belongs $\qquad 7 \notin A$ ↪ does not belong

Cardinality: The number of elements in a set is called the cardinal number, or cardinality, of the set.

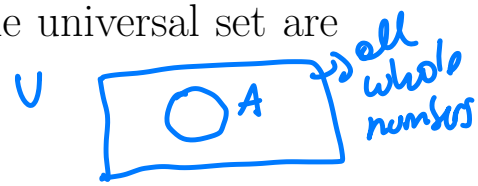
Symbol: $|A|$ → cardinality of A

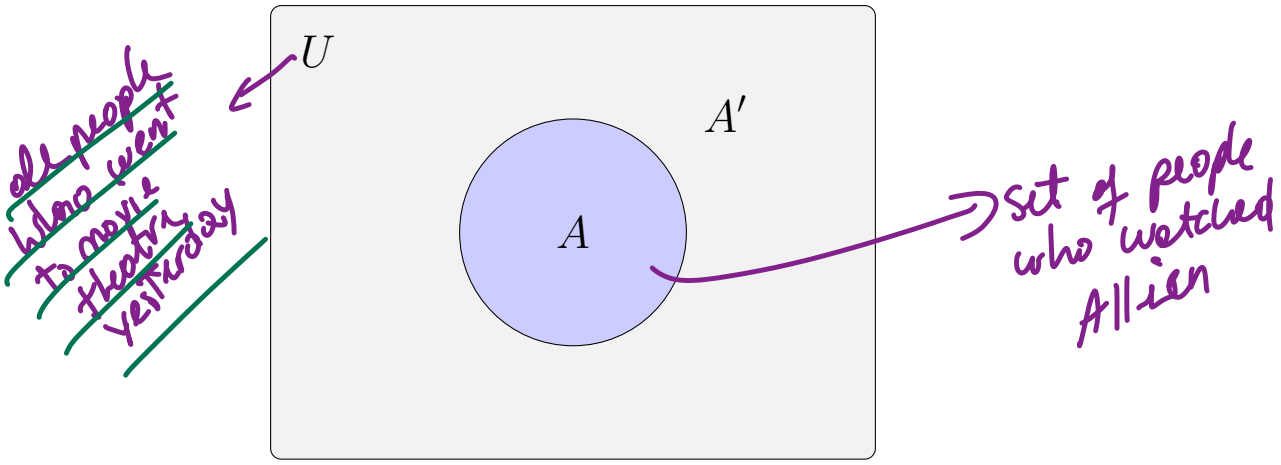
$|A| = 6$ (there are 6 elements in set A)

(cardinality $n(A)$)

Universal set: In set theory, the universe of discourse is called the Universal Set, typically designated with the letter U.

Venn Diagrams: Venn diagrams were developed by the logician John Venn (1834 – 1923). In these diagrams, the universal set is represented by a rectangle and other sets of interest within the universal set are depicted as circular regions.





Complement of a set: The complement of a set A contains everything that is NOT in the set A. The complement is notated A' or A^c. (or $\bar{A}$)

A' set of people who did not watch Alien.

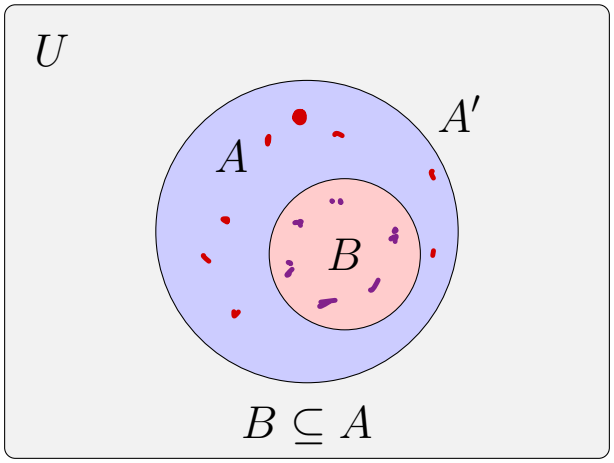
Example 2. Example: Let $A = \{2, 4, 6\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ Find A'.

$B = \{1, 3, 5, 7, 9\}$

$A' = \{1, 3, 5, 7, 8, 9, 10\}$

$B^c = \{2, 4, 6, 8, 10\}$

Subset: Set B is a subset of set A if every element of B is also an element of A. In symbols this is written as $B \subseteq A$.



$A \subseteq B$?
NO
$A \not\subseteq B$
A is not a subset of B.

Example 3. Fill in the blank with $\subseteq$ to make a true statement.

1. $\{a, b, c\} \not\subseteq \{a, c, d\}$
2. $\{1, 2, 3, 4\} \subseteq \{1, 2, 3, 4, 5, 6\}$

Difference of Sets: The difference of sets A and B, written $A - B$ is the set of elements belonging to set A and not to set B.

Example 4. Find $A - B$ if $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$

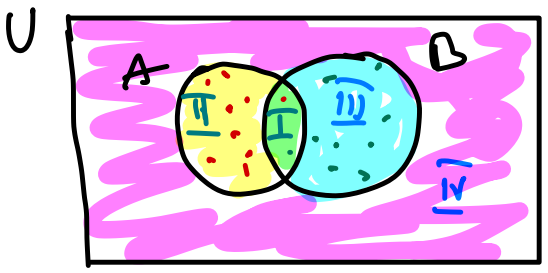
$A - B = \{1\}$

$B - A = \{4\}$

Intersection: The intersection of sets A and B, written $A \cap B$ is the set of elements common to both A and B.

$A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$

$A \cap B = \{2, 3\}$



4 Regions:
I $A \cap B$
II $A - B$
III $B - A$
IV $(A \cup B)^c$

$A \cup B \rightarrow$ II, I, III

Union: The union of sets A and B, written $A \cup B$ is the set of elements belonging to either of the sets.
This is the "inclusive or". It means one or the other or both.

$A \cup B = \{1, 2, 3, 4\}$

Example 5. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 3, 5, 7, 9\}$
$B = \{2, 4, 6\}$
$C = \{1, 2, 3\}$
Find the following:

$B' = B^c = \{1, 3, 5, 7, 8, 9, 10\}$

a) $A \cap C = \{1, 3\}$

b) $A - C = \{5, 7, 9\}$

c) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9\}$

d) $B \cap B' = \{2, 4, 6\} \cap \{1, 3, 5, 7, 8, 9, 10\} = \phi$

e) $C \cap \emptyset = \{1, 2, 3\} \cap \phi = \phi$

General Rules:
1) $A \cap A' = \phi$
2) $A \cap \phi = \phi$
3) $A \cup \phi = A$

f) $C \cup \emptyset = \{1,2,3\} \cup \emptyset = \{1,2,3\} = C$

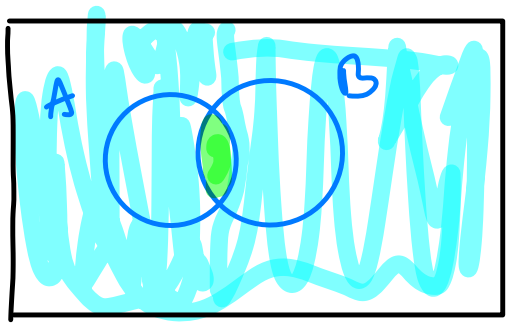
g) $(A \cup B) \cap C = \{1,2,3,4,5,6,7,9\} \cap \{1,2,3\} = \{1,2,3\} = C$

h) $(B \cup A)' = \{1,2,3,4,5,6,7,9\}' = \{8,10\}$

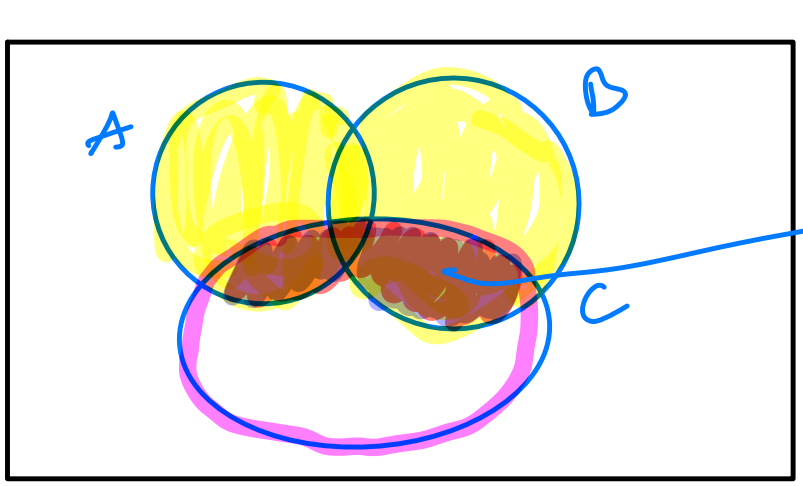
Examples with Venn Diagram

1. Draw a Venn Diagram to represent the set $(A \cap B)'$
2. Draw a Venn Diagram to represent $(A \cup B) \cap C$

1) U



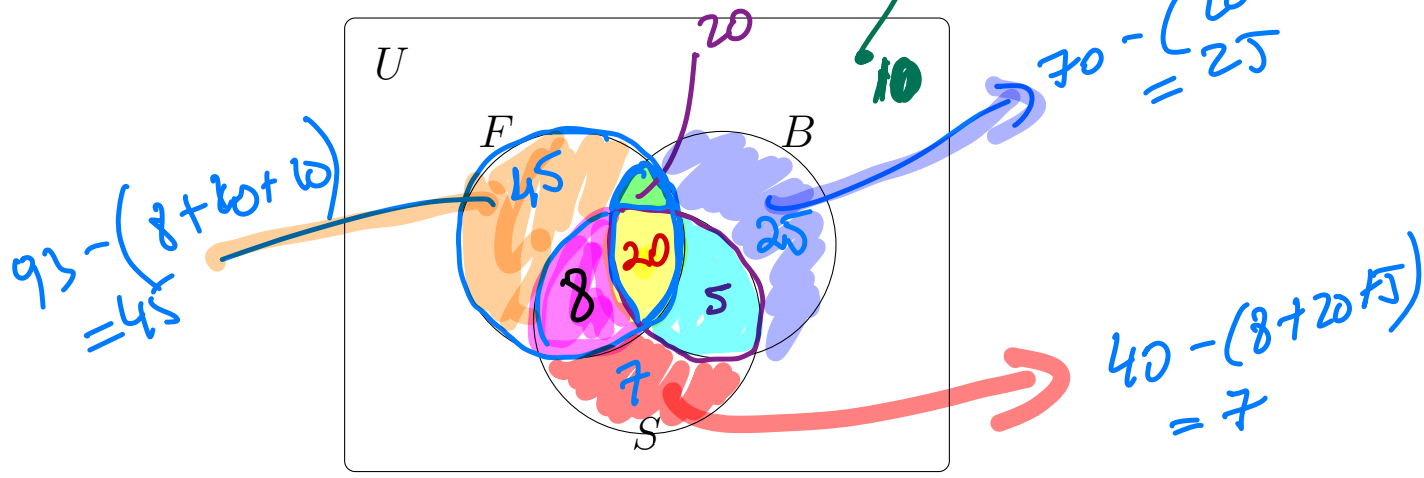
2)



Example with 3 Venn Diagrams

Example 6. Suppose that a group of 140 people were questioned about particular sports that they watch regularly and the following information was produced.

- 93 like watching football
- 70 like watching baseball
- 40 like watching soccer
- 40 like watching football and baseball
- 25 like watching baseball and soccer
- 28 like watching football and soccer
- 20 like watching all three



Questions:

a. How many people like watching only football? 45

b. How many people don't like watching any of these sports? 10