Day 11, Tuesday

HW 8 due today Parabolas
HW 9 Function due Thursday
W HW 2 due on Thursday

Exam 2 on Tuesday 11th

Ellipse 7.1
Hyperbola 7.2
Parabolas 7.3
Functions & Relations 2.3
Linear Equations & Functions 2.4 & 2.5

Day 11 Notes
Tuesday March 4th
2.4 and 2.5 Linear Equations and Linear

2.4 Linear Equations

Linear equations and linear functions form the foundation of algebra and are essential mathematical tools that model relationships where one quantity changes at a constant rate with respect to another.

1 Graphing Linear Equations in two variables

Let A, B and C be real numbers such that both A and B are not zero. A **linear equation** in the variable x and y is an equation that can be written in any of the forms:

$Ax^2 + By + x = 5$ not linear of x^2 term

standard form: $\boxed{Ax + By = C}$ *

slope-intercept form: * $\boxed{y = mx + b}$ where m is slope and b y intercept

point-slope form: Given a slope m and a point (x_1, y_1) the equation of line is * $\boxed{y - y_1 = m(x - x_1)}$

Example 1. Graph the line represented by each equation.

1. $2x + 3y = 6$ this is st. form.

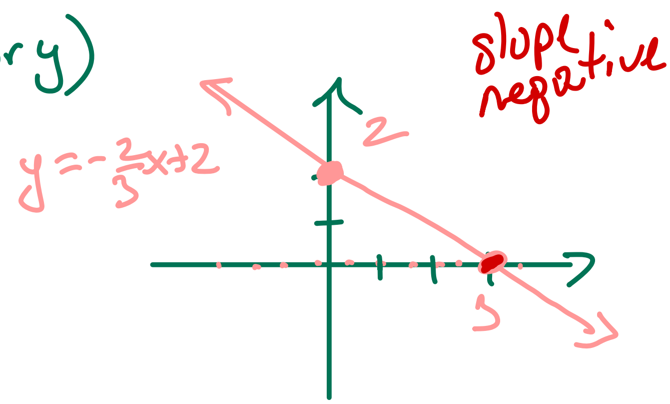
slope intercept form (solve for y)

$2x + 3y = 6$
$-2x \quad -2x$

$\frac{3y}{3} = \frac{-2x + 6}{3}$

$\boxed{y = -\frac{2}{3}x + 2}$

$m = -2/3$, $b = 2$ $(0, 2)$



x intercept: set $y = 0$
$2x + 3 \cdot 0 = 6 \Rightarrow 2x = 6$
$x = 3$

2. $2y = 4$

3. $x = -3$

(2) $2y = 4$

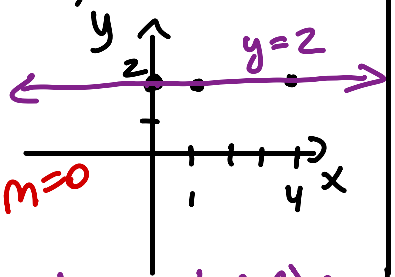
$y = 2$

$Ax + By = C$

$A = 0,\ B = 2,\ C = 4$

x	y
1	2
0	2
4	2

function



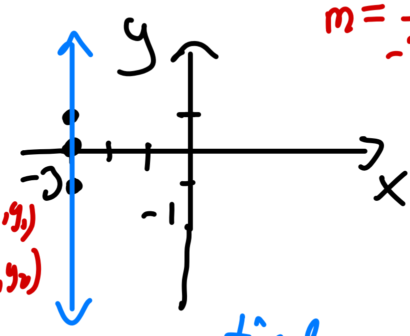
$m = 0$

horizontal line

(3) $x = -3$ → linear ?

$Ax + By = C,\ A = 1,\ B = 0,\ C = -3$

x	y
-3	-1
(-3	0) (x_1, y_1)
(-3	1) (x_2, y_2)



$m = \dfrac{1-0}{-3-(-3)} = \dfrac{1}{0}$

↓

undefined

$x = -3$ vertical line

not a function!

2 Determine the slope of a line

One of the important characteristic of a nonvertical line is that for every 1 unit increase in the x variable, the y variable change is a constant m, called the slope.

Slope Formula:

The slope of a line passing through distinct points (x_1, y_1) and (x_2, y_2) is

→ delta (means change)

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

Example 2. Find the slope of the line passing through the points $(-3, -2)$ and $(2, 5)$.

(x_1, y_1)

(x_2, y_2)

$m = ?$

$$\boxed{y = mx + b}$$

$$\boxed{y - y_1 = m(x - x_1)}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{2 - (-3)} = \frac{7}{5}$$

$y - (-2) = \frac{7}{5}(x - (-3))$

$y + 2 = \frac{7}{5}(x + 3)$ → $y = \frac{7}{5}(x + 3) - 2$

$y = \frac{7}{5}x + \frac{21}{5} - \frac{10}{5}$

$y = \frac{7}{5}x + \frac{11}{5}$

Example 3. Write the equation of the line that passes through the point $(2, -3)$ and has slope -4.

$y - y_1 = m(x - x_1)$

$y - (-3) = -4(x - 2)$

$y + 3 = -4x + 8$

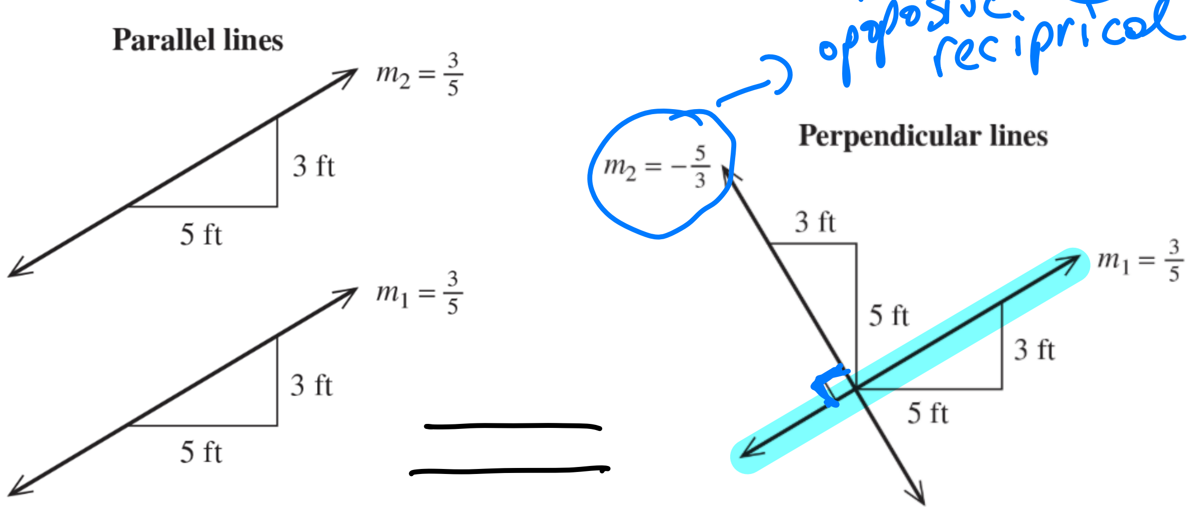
$\boxed{y = -4x + 5}$

~~$y = -\frac{4}{1}x - 11$~~

$y = -4x + 5$

3 Determine the slope of parallel and Perpendicular Lines

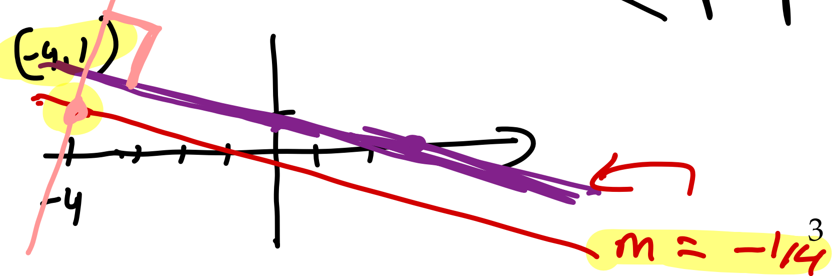
opposite in sign, reciprical



If m_1 and m_2 are the slopes of two nonvertical parallel lines, then $m_1 = m_2$.

If m_1 and m_2 are the slopes of two nonvertical perpendicular lines, then $m_1 = -\frac{1}{m_2}$ or $m_1 m_2 = -1$.

$\boxed{\text{Ex}}$ Find the equation of the line passing through $(-4, 1)$ and
- parallel to line $x + 4y = 3$. ⓐ
- perpendicular to line $x + 4y = 3$ ⓑ



$x + 4y = 3$

$4y = -x + 3$

$y = -\frac{1}{4}x + \frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 1 = -1/4(x - (-4))$

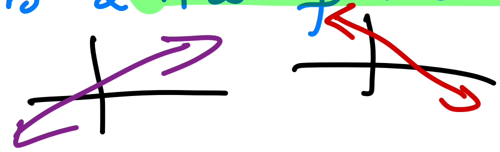
$y - 1 = -\frac{1}{4} \cdot (x + 4)$

$y - 1 = -\frac{1}{4}x - 1$

$y = -1/4 x$

Definition of Linear and Constant Functions Let m and b represent real numbers where $m \neq 0$.

- A function defined by $f(x)=mx+b$ is a linear function. Its graph is a slanted line.



- A function defined by $f(x)=b$ is a constant function. Its graph is a horizontal line.

4 Average Rate of Change

ARC

(x_1, y_1)

The **Average Rate of Change** of a function between two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is the slope of the line going through those two points.

(x_2, y_2)

$$ARC = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

ARC of a function between $(x_1, f(x_1))$ and $(x_2, f(x_2))$ $= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

Example 4. What is the ARC of $f(x) = 2x + 1$ between the points $x_1 = -1/2$ and $x_2 = 1/2$? What about between the points $x_1 = 2$ and $x_2 = 3$?

$m = 2$

$$ARC = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{2-0}{\frac{1}{2} - (-\frac{1}{2})} = \frac{2}{1} = 2$$

$f(x_2) = f(\tfrac{1}{2}) = 2 \cdot \tfrac{1}{2} + 1 = 2$

$f(x_1) = f(-\tfrac{1}{2}) = 2(-\tfrac{1}{2}) + 1 = 0$

$f(3) = 2 \cdot 3 + 1 = 7$

$f(2) = 2 \cdot 2 + 1 = 5$

$$* \; ARC = \frac{f(3) - f(2)}{3-2} = \frac{7-5}{1} = 2$$

ARC for a linear function is constant & it equals the slope.

The graphs of many function are not linear but we are still interested to see what is the average rate of change between two points.

Example 5. Finding ARC of nonliner functions

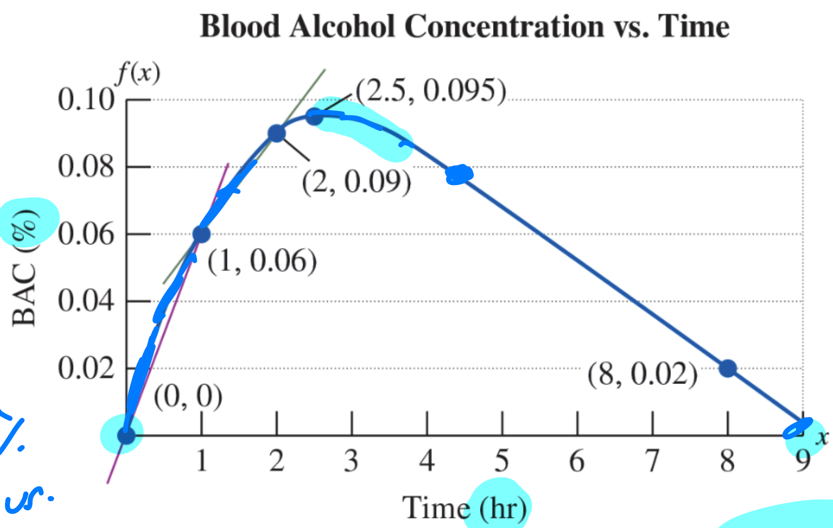
Determine the average rate of change of blood alcohol level

a. from $x_1 = 0$ to $x_2 = 1$.

b. from $x_1 = 1$ to $x_2 = 2$.

c. Interpret the results from parts (a) and (b).

c) The BAC rose by an average of 0.06% during the first hour.
The BAC rose by an average of 0.03% during second hour.



a) $$ARC = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{0.06 - 0}{1 - 0} = 0.06$$

$f(x_2) = f(1) = 0.06$

$f(x_1) = f(0) = 0$

b) $$\frac{0.09 - 0.06}{2-1} = 0.03$$

In many day-to-day applications, two variables are related linearly and thus can be modeled by a linear function.

For each of the following scenarios, create a linear function model. Be sure to:

- Identify the variables and what they represent
- Find the slope and y-intercept (or appropriate equivalent)
- Write the linear function in the form $f(x) = mx + b$
- Use your model to answer the questions

1. **Car Rental Company Pricing**

   A car rental company charges a daily rate of \$45 plus \$0.25 per mile driven.

   (a) Create a linear function that models the total cost $C(m)$ of renting a car for one day and driving m miles.

   (b) What would be the total cost of renting a car for one day and driving 120 miles? → $m=120$ cost = ?

   (c) How many miles can be driven in one day if the budget is \$~~75~~ 100?

   $m = ?$

a) input = independent variable = miles driven denoted with m

output = dependent variable = dollars / cost

$$C(m) = 45 + 0.25m$$

→ slope $= 0.25$

b) $C(120) = ?$

$$C(120) = 45 + 0.25(120) = \$75$$

c) $100 = 45 + 0.25m \Rightarrow 55 = 0.25m$

$$m = \frac{55}{0.25} = 220 \text{ mile}$$

2. **Business Revenue and Costs**

A small manufacturing business has fixed monthly costs of \$5,000 (rent, insurance, etc.) and it costs \$15 to produce each unit of their product. Each unit sells for \$35.

(a) Create a linear function $C(x)$ that represents the total monthly cost of producing x units.

(b) Create a linear function $R(x)$ that represents the total monthly revenue from selling x units.

(c) Find the break-even point (where revenue equals cost).

(d) What is the profit or loss if 400 units are produced and sold in a month?

a) $C(x) = 15x + 5000$

b) $R(x) = 35x$

c) $C(x) = R(x)$

money coming out = money coming in

$\underbrace{15x + 5000}_{C(x)} = \underbrace{35x}_{R(x)}$

$-5000 \qquad -5000$

$15x = 35x - 5000$

$-35x \qquad -35x$

$\frac{-20x}{-20} = \frac{-5000}{-20}$

$x = 250$ units

d) profit bc 400 is greater 250 units.

$P(x) \to$ profit fuction.

$P(x) =$ money coming in $-$ money coming out $= R(x) - C(x)$

$P(400) = 35(400) - (5000 + 15(400)) = \$3000$