Agenda for Wednesday, Oct 23rd 2024

Coming up

- Student Hours Tuesday today 3:45pm-5pm in C162
- HW 13 due on Monday
- Excel Assignment 3 will be posted today and due next Wednesday
- Group Quiz on Monday

Recall:

X	$P(X)$
all possible values	

mean = expected value

From previous day:

Example 1. A charity organization is selling \$5 raffle tickets as part of a fund-raising program. The first prize is a trip to Mexico valued at \$3450, and the second prize is a weekend spa package valued at \$750. The remaining 10 prizes are \$25 gift cards. The number of tickets sold is 6000. Find the expected value of a single raffle ticket.

	X (L_1)	$P(x)$ (L_2)
Mexico	3445	1/6000
spa	745	1/6000
gift card	20	10/6000
looser	0-5=-5	5988/6000

$E(X) = \mu =$ mean (average)

1 Var Stats $L_1, L_2 = -4.26$

$E(X) = -\$4.26$

Each raffle ticket has an average loss of \$4.26.

Binomial Distribution

Next, we will look into a special discrete probability distributions called **Binomial Distributions.**
There are **four conditions** that the experiment has to meet to be considered a **binomial experiment:**

- Condition 1: set amount of trials
  we denote the amount of trials
  with n
- Condition 2: each trial has 2 outcomes
  "success" or "failure"
- Condition 3: each trial is independent
- Condition 4: the probability of success, denoted
  with p, is the same for each trial.

Example 2. Are the following binomial experiment:

1. Randomly guessing at a multiple-choice question with 4 possible answers has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. Joe guesses on each question with no pattern.
   - Condition 1: $n=6$
   - Condition 2: ✓
   - Condition 3: ✓
   - Condition 4: $p=\frac{1}{4}=0.25$

   yes

prob of failure $q=1-p=1-0.25=0.75$

2. Randomly guessing at a multiple-choice question has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. The first 3 questions have 4 choices, and the last 3 questions have 5 choices each. Cesar guesses on each question with no pattern. Determine if this is a binomial experiment.

- Condition 1: ✓
- Condition 2: ✓
- Condition 3: ✓
- Condition 4: ✗

NO

3. Cyanosis is the condition of having bluish skin due to insufficient oxygen in the blood. About 80% of babies born with cyanosis recover fully. A hospital is caring for 10 babies born with cyanosis. The random variable represents the number of babies that recover fully.

- Condition 1: $n = 10$
- Condition 2: 2 outcomes ✓
- Condition 3: ✓
- Condition 4: $p = 0.8$

yes

X	
0	
1	
2	
⋮	
10	

Notation and Properties of the Binomial distribution

The outcomes of a binomial experiment fit a binomial probability distribution. We say the random variable

$X =$ the number of successes obtained in the n independent trials

is a binomial random variable and we write $\boxed{X \sim B(n, p)}$

ex: $X \sim B(10, 0.8)$

$n-k$
$10-3$

To find the probability that our random variable X takes on a certain value (denoted k) we use the following formula:

$P(x = k) = {}_nC_k \, p^k \cdot q^{n-k}$

ex: $P(x=3) =$ prob that 3 babies recover fully $= {}_{10}C_3 (0.8)^3 \cdot (0.2)^7$

The **mean** of a binomial probability distribution is

$\mu = np$ $\qquad \mu = 10(.8) = 8$

The **variance** of a binomial probability distribution is

$\sigma^2 = npq$

The **standard deviation** is

$\sigma = \sqrt{npq}$

$\boxed{X \sim B(6, .25)}$

$p = \frac{1}{4} = 0.25$

Example 3. Randomly guessing at a multiple-choice question with 4 possible answers has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. Mayra guesses on each question with no pattern.

$n = 6$

1. Find the probability of Mayra guessing correctly exactly 3 questions.

$P(x=3) = \text{binomialpdf}(n, p, k) = \text{binomialpdf}(6, .25, 3)$
$= .132$

2. Find the probability of Mayra guessing correctly all of the questions.

$P(x=6) = \text{binomialpdf}(6, .25, 6) = .000244$

3. Find the mean or expected number of questions Mayra guesses correctly.

$\mu = np = 6(.25) = 1.5$

4. Find the standard deviation of the correct guesses.

$\sigma = \sqrt{npq} = \sqrt{6 \cdot (.25)(.75)} = 1.061$

5. Create the binomial probability distribution.

all values of the random variable X	corresponding probabilities: P(X)
$P(X=0)$	binomialpdf(6,.25,0) = .178
$P(X=1)$	binomialpdf(6,.25,1) = .356
$P(X=2)$	binomialpdf(6,.25,2) = .296
$P(X=3)$	.132
$P(X=4)$	binomialpdf(6,.25,4) = .033
$P(X=5)$	binomialpdf(6,.25,5) = .00439
$P(X=6)$	.000244

6. Find the probability of Mayra guessing correctly at most 3 questions.

$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$
$= .962$

Binomial distribution with calculator and Excel

There two functions in the calculator that calculate binomial probabilities. Press 2nd then **vars** to get to Distr

followed by


The two functions are:

- binomialpdf(n, p, k) calculates probabilities of the form $P(X=K)$
- binomialcdf(n, p, k) = binomialcdf(6, .25, 3) = .962
  cumulative
  for probabilities of the form $P(X \le K)$.

Excel Instructions:

- Type =BINOM.DIST(k, n, p, FALSE) to find probabilities of the type $P(x = k)$.
- Type =BINOM.DIST(k, n, p, TRUE) to find probabilities of the type $P(x \leq k)$.

check your answers

Example 4. It has been stated that about 41% of adult workers have a high school diploma but do not pursue any further education. 20 adult workers are randomly selected. $p = .41$, $n = 20$

1. Find the probability that **exactly** 2 of them have a high school diploma but do not pursue any further education.

   $P(x=2) = \text{binomialpdf}(20, .41, 2)$

2. Find the probability that **at most** 12 of them have a high school diploma.

   $P(x \leq 12) = \text{binomialcdf}(20, .41, 12)$

   means 12 and above

3. Find the probability that **at least** 6 of them have a high school diploma.

   $P(x \geq 6)$ means 6 and above $=$ all of them add to 1 $- P(x \leq 5) = 1 - \text{binomialcdf}(20, .41, 5)$

4. Find the probability that **more than** 10 of them have a high school diploma.

   $P(x > 10) = 1 - P(x \leq 10) = 1 - \text{binomialcdf}(20, .41, 10)$

   mean 11 and above

5. Find the probability that **less** 5 of them have a high school diploma.

   $P(x < 5) = \text{binomialcdf}(20, .41, 4)$

   means 4 and below

6. How many adult workers do you expect to have a high school diploma but do not pursue any further education?

   $\mu = np = (20)(.41)$