

Total Possible Points: 118

Name: _____

1. Find the vertex of the parabola by applying the vertex formula. (4)

$$f(x) = -\frac{1}{2}x^2 + 6x + 9.$$

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2. Consider the following quadratic function. (4)

$$f(x) = 25x^2 - 10x + 9$$

- (a) Find the discriminant.

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- (b) Based on the discriminant, how many x-intercepts does the graph of f have?

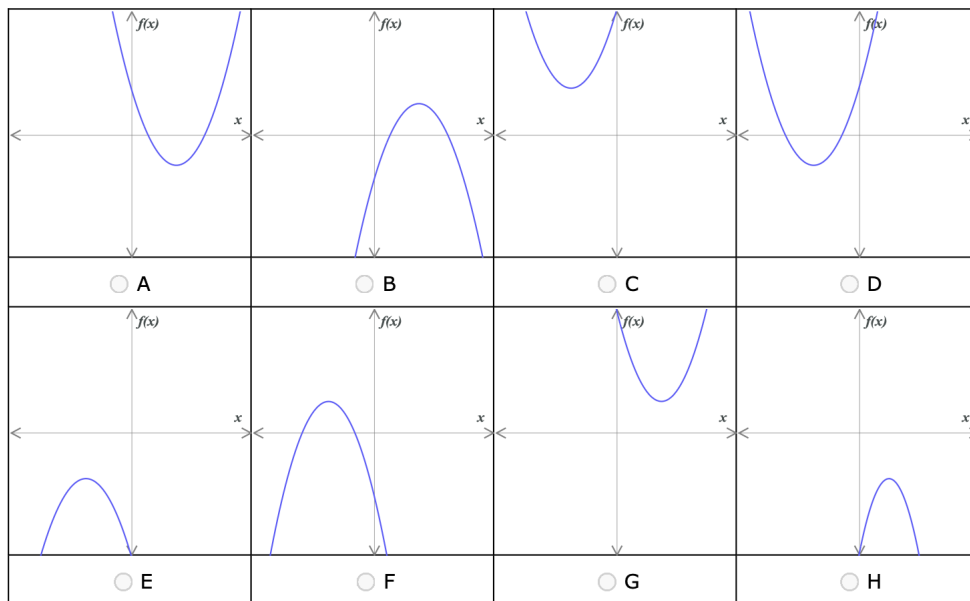
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3. Find the zeros of the function and state the multiplicities. (3)

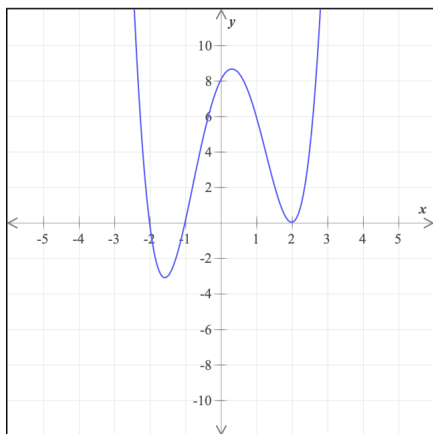
$$t(x) = x^3 + 2x^2 - 16x - 32$$

4. Given a quadratic function defined by $f(x) = a(x - h)^2 + k$, ($a \neq 0$), (3)
match the graph with the function based on the conditions given.

$a < 0$, axis of symmetry $x = 2$, $k > 0$



5. Consider the following graph of a polynomial. (3)



- Determine the minimum degree of the polynomial.
- Determine whether the
 - leading coefficient is positive or negative
 - degree of the polynomial is odd or even.

6. Consider the function.

(8)

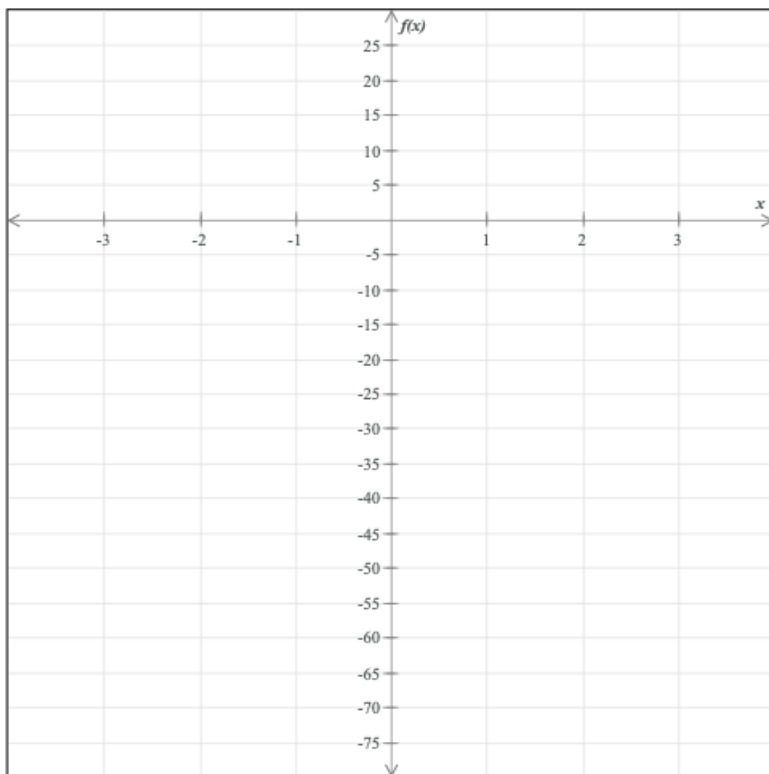
$$f(x) = -x^4 + 15x^2 - 54$$

(a) State the end behavior of the function.

(b) Find y-intercept(s).

(c) Find the real zero(s) of the function.

(d) Sketch the function.



7. Use long division to divide. Express any numbers as integers or simplified fractions. (5)

$$(x^5 + 2x^4 + 12x^2 - 5x + 16) \div (x^2 + 2)$$

8. Given a polynomial $f(x)$, the quotient $\frac{f(x)}{x-8}$ has a remainder of 25. What is the value of $f(8)$? (3)

9. Use the factor theorem to determine if $x-4$ is a factor of $f(x) = x^3 + 64$. (3)

10. List the possible rational zeros. (4)

$$n(x) = -4x^5 - 13x^3 + 6x^2 - 10$$

11. If $f(x)$ is a polynomial with real coefficients and zeros of 5 (multiplicity 1), -5 (multiplicity 1), $-5 + 7i$, and $4 - 3i$, what is the minimum degree of $f(x)$? (3)

12. A polynomial $f(x)$ and one or more of its zeros are given. (6)

$$f(x) = x^4 - 16x^3 + 78x^2 + 32x - 160$$

$8 - 4i$ is a zero and $\sqrt{2}$ is a zero.

- (a) Find all the zeros. Write the answer in exact form.

- (b) Factor $f(x)$ as a product of linear factors.

13. Consider the following rational function (6)

$$f(x) = \frac{(x+3)(1-6x)}{(5x+1)(x+3)}.$$

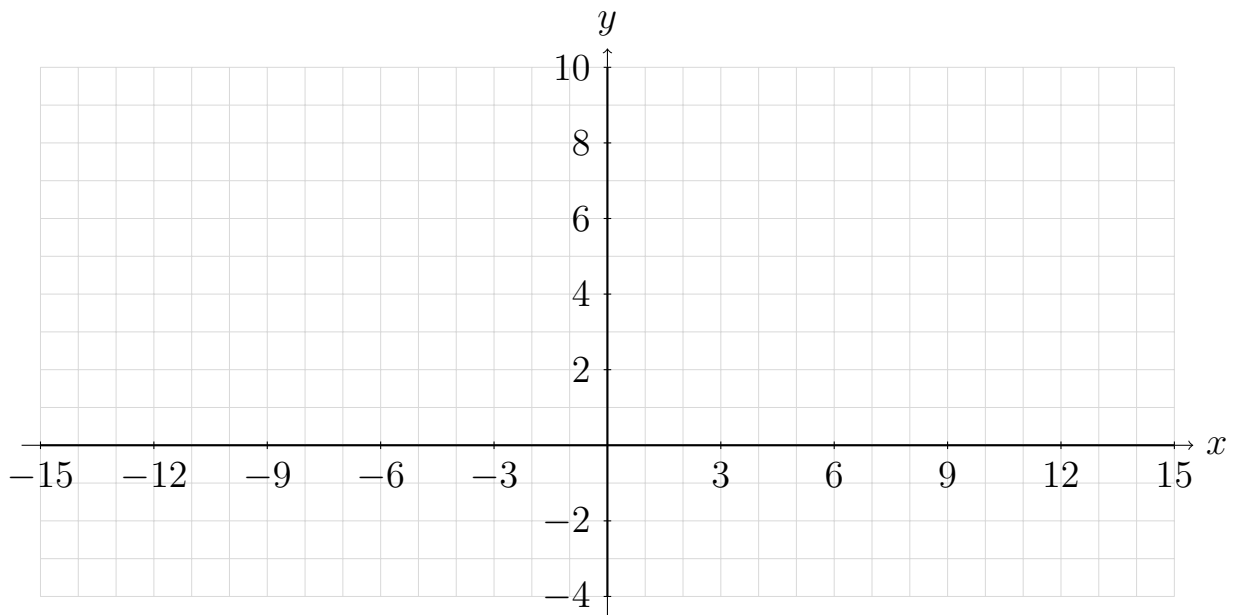
- (a) Does the rational function have a hole? If yes, give the x and y coordinate of the hole?

- (b) Does the rational function have horizontal asymptote? If yes, give the equation of it.

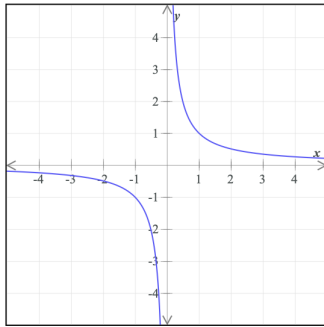
- (c) Does the rational function have vertical asymptote(s)? If yes, give the equation(s) of it.

14. Graph the following rational function $f(x) = \frac{x^3 - x^2 - 2x}{x^3 + 5x^2 + 6x}$. (8)

To receive full points you must label (if any exists) holes, x and y intercepts and asymptotes.

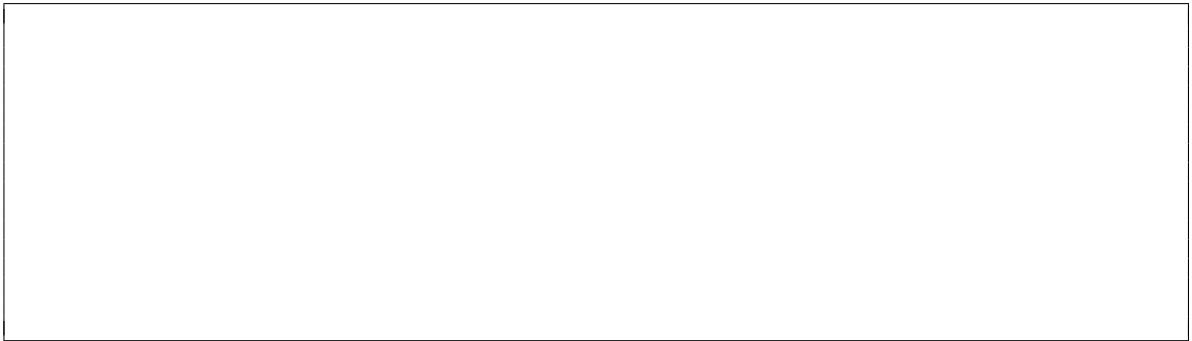


15. Determine if the relation defines y as a one-to-one function of x . (3)

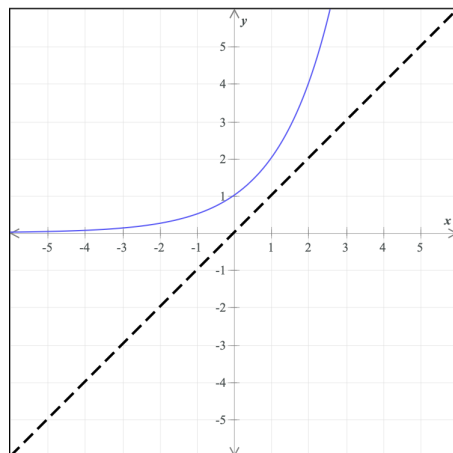


16. A one-to-one function is given. Write an equation for the inverse function. (5)

$$f(x) = \frac{x - 4}{x + 7}$$



17. The graph of a function is given. Graph the inverse function. (3)



18. A function g is given. Identify the parent function. Then use the steps for graphing multiple transformations of functions to list, in order, the transformations applied to the parent function to obtain the graph of g . (5)

$$g(x) = \frac{4}{x+2} - 3$$

19. Write a function based on the given parent function and transformations in the given order. (5)

Parent function: $y = |x|$

1. Shift 4 units to the right.
2. Shrink horizontally by a factor of $\frac{1}{4}$.
3. Reflect across the y -axis.

20. Given $f(x) = x^2 - 7x$, find the difference quotient $\frac{f(x+h) - f(x)}{h}$ and simplify. (6)

21. Given the functions:

(8)

$$f(x) = -7x$$

$$g(x) = |x - 5|$$

$$h(x) = \frac{1}{x + 6}$$

Answer the following:

(a) $\left(\frac{f}{g}\right)(-5)$

(b) $f \circ g$

(c) $g \circ f$

(d) Domain of $(f \circ h)(x)$

22. Consider a right circular cone with given height h . The volume of the cone as a function of its radius r is given by $V(r) = \frac{1}{3}\pi r^2 h$. (6)

Consider a right circular cone with fixed height $h = 10$ in.

(a) Write the diameter d of the cone as a function of the radius r .

(b) Write the radius r as a function of the diameter d .

(c) Find $(V \circ r)(d)$ and interpret its meaning. Assume that $h = 10$ in. Write your answer as a fraction if necessary.

23. Factor the following.

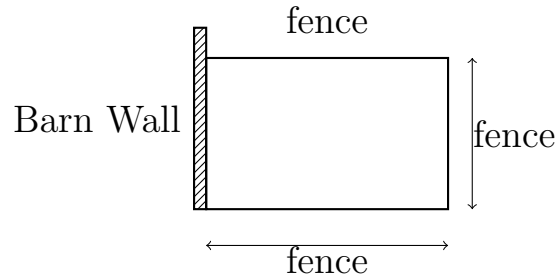
(4)

(a) $49n^2 - 81$

(b) $x^2 - x - 90$

(c) $3x^2 - 8x - 3$

24. A farmer has 240 feet of fencing and wants to enclose a rectangular area for grazing cattle. One side of the rectangle will be against an existing barn wall, so fencing is only needed for three sides.



- (a) Let x represent the width of the rectangle (perpendicular to the barn wall) and y represent the length (parallel to the barn wall). Express the area A of the rectangle as a function of x only.
- (c) What are the dimensions of the rectangle that will maximize the enclosed area? Show your work.
- (d) What is the maximum area that can be enclosed? (1 point)

25. Consider the given function $q(x) = 2x^2 - 8x + 7$. (10)

(a) Write the function in vertex form.

(b) Identify the vertex.

(c) Determine the x -intercept(s).

(d) Determine the y -intercept(s).

(e) Determine the axis of symmetry.

(f) Determine the minimum or maximum value of the function.

(g) Write the domain and range in interval notation.