

1. Sequences

In this section we are going to introduce Sequences and Series which are fundamental mathematical concepts that are important for understanding patterns and are used in practice to model population growth in biology and analyze algorithms in computer science.

Definition. Roughly speaking, a **sequence** is an

Example 1. A simple example is the sequence:

$$5, \quad 10, \quad 15, \quad 20, \quad \dots$$

The numbers in the sequence are called _____ and are often written as $a_1, a_2, a_3, \dots$. The dots mean that the list continues forever.

Notation:

$$a_2$$

$$a_n$$

$$\{a_n\}$$

We can describe the pattern of the sequence displayed above by a formula for the general term:

Example 2. Find the first three terms and the 100th term of the sequence defined by each formula.

1. $a_n = 2n - 1$

3. $c_n = \frac{n}{n+1}$

$\{a_n\} =$

$\{c_n\} =$

2. $b_n = n^2 - 1$

4. $d_n = \frac{(-1)^{n+1}}{n}$

$\{b_n\} =$

$\{d_n\} =$

Example 3. Find the general term of a sequence whose first several terms are given.

(a) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$

(b) $-2, 4, -8, 16, -32, \dots$

Definition. An **alternating sequence** is a sequence with

2. Recursively defined sequences

Some sequences do not have simple defining formulas like those of the preceding example.

Definition. When the n th term of a sequence may depend on some or all of the terms preceding it we call the sequence _____.

Example 4. Example of recursive sequence:

$$a_1 = 4 \text{ and } a_n = 2a_{n-1} + 1 \text{ for } n > 1.$$

Example 5. The Fibonacci Sequence:

Find the first 7 terms of the sequence defined recursively by

$$a_1 = 1, a_2 = 1 \text{ and } a_n = a_{n-1} + a_{n-2}.$$

3. Factorial notation

Factorials form the basis for important series like Taylor series expansions.

Definition. The factorial of a positive integer n , written as $n!$, is

For example, $5! =$

$10! =$

$n! =$

By definition, $0! =$

Example 6. Evaluate the following expressions.

(a) $\frac{8!}{3! \cdot 5!}$

(b) $\frac{n!}{(n+2)!}$

Example 7. Given the sequence defined by $b_n = \frac{n^2}{(n+1)!}$ find b_1 and b_6 .