Math 142: Wednesday, Nov 6th, Day 22 Notes

Agenda for Wednesday, Nov 6th 2024

Coming up

- Student Hours Thursday 2:15-3:30pm in C162
- HW 15 due today
- Excel Assignment 4 posted
- Priority Spring Registration open

Estimating population parameters using sample statistics

population
ex: all CLC

$\mu = ?$
$\sigma = 3$ inches

inferential statistic

sample of CLC students
of n

$\bar{X}$ = sample mean
S = sample st. dev.

μ is mean heigth of all CLC students

σ is st. dev

ex: $\bar{X}_1 = 66.7$ inches

$\bar{X}_2 = 66.56$ inches

$\bar{X}_1$ is called a point estimate for μ

We also can estimate μ with a range of values

ex: $(66, 68) \rightarrow$ interval notation

What is a confidence interval

A confidence interval for a mean provides a range of values that likely contains the true population mean, μ, along with a **confidence level**.

(lower bound , upper bound)

(66, 68)

lower bound $< \mu <$ upper bound

$66 < \mu < 68$

$\bar{x} -$ margin of error $< \mu < \bar{x} +$ margin of error

$\bar{x} - ME < \mu < \bar{x} + ME$

$ME = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

st. error

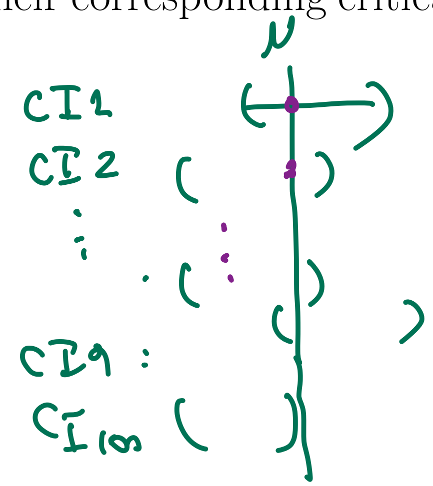
$\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

critical value

Confidence interval $\boxed{\left(\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right)}$

Confidence level (CL): it is usually expressed as a percentage. The most common used confidence levels are 90%, 95%, 99%. Their corresponding critical values are:

- For 90% CI: $z_{\alpha/2} = 1.645$
- For 95% CI: $z_{\alpha/2} = 1.96$
- For 99% CI: $z_{\alpha/2} = 2.576$



Interpretation of the confidence interval

"If we repeated this sampling process many times and calculated the confidence interval each time, about 95% of these intervals would contain the true population mean"

Confidence interval for for mean when σ is known

Formula for a confidence interval for a population mean when σ is known, is:

$$\text{CI} = \bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

where

How to calculate a confidence interval

Step 1: Decide what confidence level you are asked to use

Step 2: Find the corresponding critical value $z_{\alpha/2}$:

For 90% CI: use $z_{\alpha/2} = 1.645$

For 95% CI: use $z_{\alpha/2} = 1.96$

For 99% CI: use $z_{\alpha/2} = 2.576$

Step 3: Calculate standard error (SE):

$$SE = \frac{\sigma}{\sqrt{n}}$$

Step 4: Calculate margin of error (ME):

$$ME = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Step 5: Calculate interval bounds:

$$\textbf{Lower bound} = \bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\textbf{Upper bound} = \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Step 6: Interpret the confidence interval

Example 1. A company claims their batteries last 100 hours on average. In a sample of 36 batteries, the mean lifetime was 95 hours. The population standard deviation is known to be 12 hours. Calculate a 95% confidence interval for the mean battery life.

$\mu = 100$

$\sigma = 12$ $\bar{x} = 95$

$n = 36$

Step 1: Decide what confidence level you are asked to use

95%

Step 2: Find the corresponding critical value $z_{\alpha/2}$:

For 90% CI: use $z_{\alpha/2} = 1.645$

For 95% CI: use $z_{\alpha/2} = 1.96$ $\rightarrow$ use this one

For 99% CI: use $z_{\alpha/2} = 2.576$

Step 3: Calculate standard error (SE):

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{36}} = \frac{12}{6} = 2$$

Step 4: Calculate margin of error (ME):

$$ME = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = (1.96)(2) = 3.92$$

Step 5: Calculate interval bounds:

$$\textbf{Lower bound} = \bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 95 - 3.92 = 91.08$$

$$\textbf{Upper bound} = \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 95 + 3.9 = 98.92$$

Step 6: Interpret the confidence interval

CI: (91.08, 98.92)

We are 95% confident that the true average battery life is between 91.08 hours and 98.92 hours.