Exam 2 Study Guide

Functions and Trigonometry

1 Section 4.3: Right Triangle Trigonometry

1.1 Key Topics to Master:

- Definition of the six trigonometric functions in right triangles
- Relationship between sides and trigonometric functions
- Evaluating trigonometric functions using right triangles
- Using the Pythagorean Theorem to find missing sides
- Special right triangles (30°-60°-90° and 45°-45°-90°)

1.2 Essential Formulas:

- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin\theta}{\cos\theta}$
- $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin\theta}$
- $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{\cos\theta}$
- $\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{\tan\theta}$
- Pythagorean Theorem: $c^2 = a^2 + b^2$

2 Section 4.4: Trigonometric Functions of Any Angle

2.1 Key Topics to Master:

- Angles in standard position
- Reference angles for the four quadrants
- Determining signs of trigonometric functions in different quadrants
- Finding exact values using reference angles
- Trigonometric functions of real numbers (radian measure)

2.2 Essential Formulas:

- Reference angle formulas:
  - Quadrant I ($0°$ to $90°$): $\theta' = \theta$
  - Quadrant II ($90°$ to $180°$): $\theta' = 180° - \theta$
  - Quadrant III ($180°$ to $270°$): $\theta' = \theta - 180°$
  - Quadrant IV ($270°$ to $360°$): $\theta' = 360° - \theta$
- For any point (x, y) on the terminal side of angle θ with $r = \sqrt{x^2 + y^2}$:
  - $\sin\theta = \frac{y}{r}$
  - $\cos\theta = \frac{x}{r}$
  - $\tan\theta = \frac{y}{x}$ $(x \neq 0)$
  - $\csc\theta = \frac{r}{y}$ $(y \neq 0)$
  - $\sec\theta = \frac{r}{x}$ $(x \neq 0)$
  - $\cot\theta = \frac{x}{y}$ $(y \neq 0)$

3 Section 4.5: Graphs of Sine and Cosine

3.1 Key Topics to Master:

- Basic shapes of sine and cosine graphs
- Amplitude, period, and phase shift
- Vertical shift of trigonometric functions
- Graphing transformations of sine and cosine
- Domain and range of sine and cosine functions
- Key points on sine and cosine graphs

3.2 Essential Formulas:

- $y = A\sin(Bx - C) + D$
  - Amplitude: $|A|$
  - Period: $\frac{2\pi}{|B|}$
  - Phase shift: $\frac{C}{B}$
  - Vertical shift: D
- $y = A\cos(Bx - C) + D$

  - Amplitude: $|A|$
  - Period: $\frac{2\pi}{|B|}$
  - Phase shift: $\frac{C}{B}$
  - Vertical shift: D

4 Section 4.7: Inverse Trigonometric Functions

4.1 Key Topics to Master:

- Restricted domains for inverse trigonometric functions
- Notation: $\sin^{-1}(x)$, $\arcsin(x)$, etc.
- Domains and ranges of inverse trigonometric functions
- Evaluating inverse trigonometric expressions
- Compositions of trigonometric and inverse trigonometric functions
- Applications of inverse trigonometric functions

4.2 Essential Formulas:

- Domain and range restrictions:
  - $\sin^{-1}(x)$: Domain $[-1, 1]$, Range $[-\frac{\pi}{2}, \frac{\pi}{2}]$
  - $\cos^{-1}(x)$: Domain $[-1, 1]$, Range $[0, \pi]$
  - $\tan^{-1}(x)$: Domain $(-\infty, \infty)$, Range $(-\frac{\pi}{2}, \frac{\pi}{2})$
- Inverse function properties:
  - $\sin(\sin^{-1}(x)) = x$ for $-1 \leq x \leq 1$
  - $\sin^{-1}(\sin(\theta)) = \theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
  - $\cos(\cos^{-1}(x)) = x$ for $-1 \leq x \leq 1$
  - $\cos^{-1}(\cos(\theta)) = \theta$ for $0 \leq \theta \leq \pi$
  - $\tan(\tan^{-1}(x)) = x$ for all real x
  - $\tan^{-1}(\tan(\theta)) = \theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

5 Section 5.1: Using Fundamental Trigonometric Identities

5.1 Key Topics to Master:

- Reciprocal identities
- Quotient identities
- Pythagorean identities
- Even-odd properties
- Simplifying trigonometric expressions
- Rewriting trigonometric expressions
- Trigonometric substitution

5.2 Essential Formulas:

- Reciprocal identities:
  - $\sin\theta = \frac{1}{\csc\theta}$
  - $\cos\theta = \frac{1}{\sec\theta}$
  - $\tan\theta = \frac{1}{\cot\theta}$
  - $\csc\theta = \frac{1}{\sin\theta}$
  - $\sec\theta = \frac{1}{\cos\theta}$
  - $\cot\theta = \frac{1}{\tan\theta}$
- Quotient identities:
  - $\tan\theta = \frac{\sin\theta}{\cos\theta}$
  - $\cot\theta = \frac{\cos\theta}{\sin\theta}$
- Pythagorean identities:
  - $\sin^2\theta + \cos^2\theta = 1$
  - $1 + \tan^2\theta = \sec^2\theta$
  - $1 + \cot^2\theta = \csc^2\theta$
- Even-odd properties:
  - $\sin(-\theta) = -\sin\theta$ (odd function)
  - $\cos(-\theta) = \cos\theta$ (even function)

- $\tan(-\theta) = -\tan\theta$ (odd function)
- $\csc(-\theta) = -\csc\theta$ (odd function)
- $\sec(-\theta) = \sec\theta$ (even function)
- $\cot(-\theta) = -\cot\theta$ (odd function)

- Cofunction identities:
  - $\sin\theta = \cos(\frac{\pi}{2} - \theta)$
  - $\cos\theta = \sin(\frac{\pi}{2} - \theta)$
  - $\tan\theta = \cot(\frac{\pi}{2} - \theta)$
  - $\cot\theta = \tan(\frac{\pi}{2} - \theta)$
  - $\sec\theta = \csc(\frac{\pi}{2} - \theta)$
  - $\csc\theta = \sec(\frac{\pi}{2} - \theta)$

6 Section 5.2: Verifying Trigonometric Identities

6.1 Key Topics to Master:

- Understanding the difference between identities and equations
- Strategies for verifying identities
- Working with one side of the equation at a time
- Using fundamental identities to simplify expressions
- Converting all functions to sines and cosines
- Finding common denominators
- Factoring trigonometric expressions

6.2 Essential Guidelines:

- Never perform operations that assume the identity is true
- Work with the more complex side first
- Convert to sines and cosines when dealing with multiple functions
- Look for patterns that match fundamental identities
- Factor expressions when possible
- Find common denominators when dealing with fractions

7 Section 5.3: Solving Trigonometric Equations

7.1 Key Topics to Master:

- Basic strategies for solving trigonometric equations
- Using factoring to solve trigonometric equations
- Using substitution to convert to algebraic form
- Equations involving multiple angles
- Finding all solutions in a given interval
- Using identities to solve equations

7.2 Essential Strategies:

- Isolate the trigonometric function
- Apply inverse trigonometric functions
- Factor when possible
- Use substitution for quadratic forms
- Check for extraneous solutions
- Consider periodicity for general solutions

8 Section 5.4: Sum and Difference Formulas

8.1 Key Topics to Master:

- Derivation and application of sum and difference formulas
- Evaluating exact values using sum and difference formulas
- Proving identities using sum and difference formulas
- Solving equations using sum and difference formulas

8.2 Essential Formulas:

- $\sin(u+v) = \sin u \cos v + \cos u \sin v$
- $\sin(u-v) = \sin u \cos v - \cos u \sin v$
- $\cos(u+v) = \cos u \cos v - \sin u \sin v$
- $\cos(u-v) = \cos u \cos v + \sin u \sin v$
- $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$
- $\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$

9 Section 5.5: Multiple-Angle Formulas

9.1 Key Topics to Master:

- Double-angle formulas
- Power-reducing formulas
- Half-angle formulas
- Application in solving equations
- Application in evaluating expressions
- Application in calculus

9.2 Essential Formulas:

- Double-angle formulas:
  - $\sin(2\theta) = 2\sin\theta\cos\theta$
  - $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
  - $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$
- Power-reducing formulas:
  - $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$
  - $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$
  - $\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$
- Half-angle formulas:
  - $\sin(\frac{\theta}{2}) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
  - $\cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
  - $\tan(\frac{\theta}{2}) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

10 Summary of Key Concepts

1. Right triangles provide the foundation for trigonometric functions through the relationship of sides.
2. Angles in standard position extend trigonometric functions beyond acute angles.
3. Reference angles help find values for angles in all quadrants.
4. Graphs of sine and cosine have period 2π and can be transformed.
5. Inverse trigonometric functions require restricted domains to ensure they are functions.
6. Fundamental identities include reciprocal, quotient, and Pythagorean relationships.
7. Verifying identities requires strategic manipulation of expressions.
8. Trigonometric equations can be solved using algebraic techniques and understanding periodicity.
9. Sum and difference formulas relate trigonometric functions of sums and differences of angles.
10. Multiple-angle formulas include double-angle, power-reducing, and half-angle relationships.