Math 140: Chapter 5 Logic

Logical Connectives

The words **"and", "or", "if...then", "if and only if"** are examples of logical connectives. They are words that can be used to connect two simple statements to form a more complicated compound statement.
Example of compound statement:

- "If I pass Mth 140 and I pass Mth 122 then my liberal studies math requirement will be fulfilled."

Equivalent Statements

Any two statements p and q are logically equivalent if

This means that p and q will always have the same truth value, in any conceivable situation.
Examples:

- "Today I have math class and today is Saturday" is equivalent to

- "I have a dog or I have a cat" is equivalent to

Logical equivalence is denoted by the symbol $\equiv$

The Conjunction and The Disjunction

The Conjunction

If p, q are statements, their conjunction is the statement “p and q.” It is denoted by

Example:
Let p: “The number 2 is even.”
Let q: “The number 3 is odd”
Then $p \wedge q$ is a new statement
r:

Now consider these statements.
s: The number 1 is even and the number 3 is odd.
t: The number 2 is even and the number 4 is odd.
v : The number 3 is even and the number 2 is odd.

In general, in order for $p \wedge q$ to be true:

This is summarized in the following table, called a truth table.

p	q	$p \wedge q$

The Disjunction

If p, q are statements, their disjunction is the statement "p or q." It is denoted:

Consider the following four statements.
q : The number 2 is even or the number 3 is odd.
r : The number 1 is even or the number 3 is odd.
s : The number 2 is even or the number 4 is odd.
t : The number 3 is even or the number 2 is odd.

In order for a disjunction to be true,

The only time a disjunction is false is when

This is summarized in the following table, called a truth table.

p	q	$p \vee q$

Basic Rules for Constructing Truth Tables

1. The number of rows depends on the number of simple statements:
    - 1 statement— 2 rows
    - 2 statements — 4 rows
    - 3 statements — 8 rows

- 4 statements — 16 rows

2. The number of columns depends on:
   - One column for each basic statement
   - One column for each logical connective
3. The beginning columns show every possible combination of true/false
4. Each additional column is computed from previous columns

Example 1. Make a truth table for the following statement:
$(p \wedge \sim q) \vee \sim p$

Using truth tables to test for logical equivalency

To determine if two statements are equivalent, make a truth table having a column for each statement. If the columns are identical, then the statements are equivalent.

Example 2. (DeMorgan's Laws) Show these two important logical equivalencies:

1. $\sim (p \wedge q) \equiv \sim p \vee \sim q$
2. $\sim (p \vee q) \equiv \sim p \wedge \sim q$

These tell us:

- The negation of "p and q" is "not p or not q"
- The negation of "p or q" is "not p and not q"

Example 3. Use DeMorgan's Laws to write the negation of each statement:

1. I want a car and a motorcycle.
2. My cat stays outside or it makes a mess.
3. I've fallen and I can't get up.
4. You study or you don't get a good grade.

Example 4. Make a truth table for $q \vee \sim (\sim p \wedge q)$.

A **tautology** is a statement that cannot possibly be false, due to its logical structure, aka it is always true. A **contradiction** is a statement that is always false.