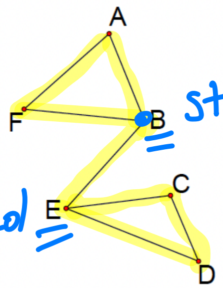
Section 1.2 Euler Paths and Circuits

Euler Path is a path that passes through every edge of a graph exactly one.

Example 1. Find an Euler path in the following graph.



start

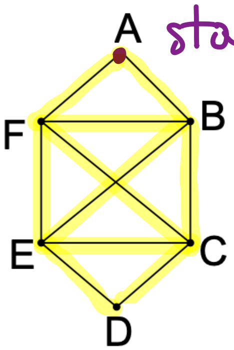
end

BAFBEDCE → Euler path

path: AFBE

Euler Circuit is a circuit → starts and ends @ same vertex. that passes through every edge of a graph exactly one.

Example 2. Find an Euler circuit in the following graph.



start & end

AFEDCEBFCBA

Try to find a different circuit.

Euler's Circuit Theorem

- If a graph is connected and every vertex has even degree, then it has at least one Euler Circuit.

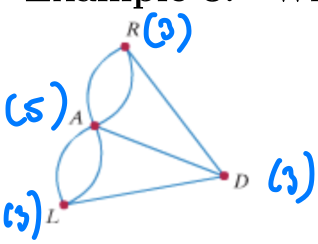
all vertices even degree they yes EC.

- If a graph has any vertices with odd degree then it will have no EC.

Euler's Path Theorem

- If a graph is connected and has exactly two odd degree vertices, then it has at least one Euler Path. The path must start at one of the odd degree vertices and end at the other. odd degree vertex.
- If a graph has more than two odd degree vertices, then it cannot have an Euler Path.

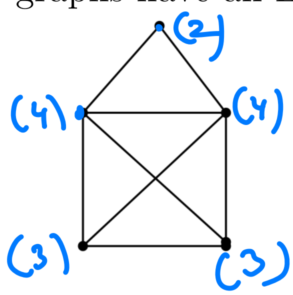
Example 3. Which of these graphs have an Euler's circuit or an Euler's Path?



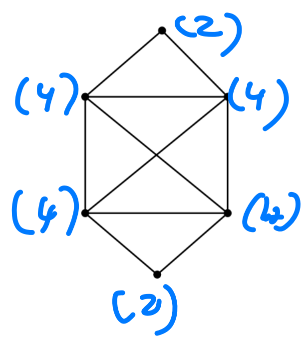
all odd degrees

no EC

no EP



has EP



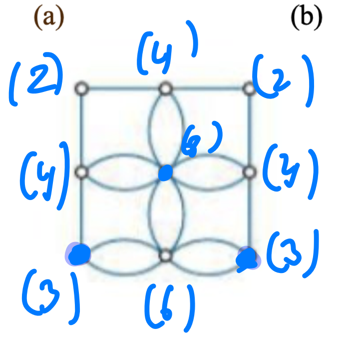
all vertices have even degree

has EC

Example 4. For each of the three graphs below choose from one of the following answers and provide a short explanation for your answer.

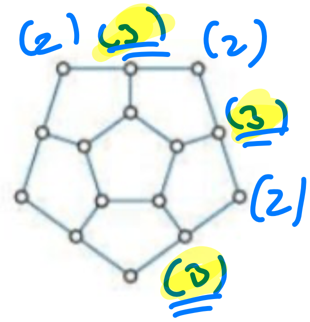
1. The graph has an Euler circuit.
2. The graph has an Euler path.
3. The graph has neither an Euler circuit nor an Euler path.
4. The graph may or may not have an Euler circuit.
5. The graph may or may not have an Euler path.

(a)



2) has EP

(b)



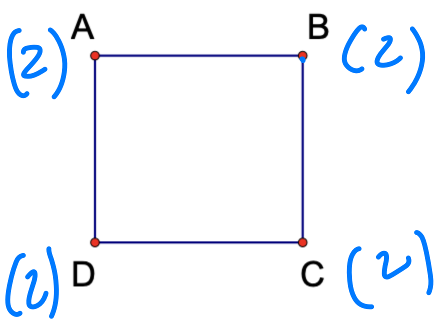
3) none
no EC
no EP

(c) A graph with eight vertices: six vertices of degree 2 and two vertices of degree 3.

5) may or may not have EP

(it depends if it's connected)

Example 5. Is it possible to add an edge so that you get just one odd vertex?



not possible!

Euler's Sum of Degrees Theorem

- The sum of the degrees of all the vertices of a graph equals twice the number of edges.

ex 5: edges : 4 degrees 2+2+2+2 = 8

8 = 2(4)

- A graph has always an even number of odd vertices.

(0, 2, 4, 6, ... odd vertices

Now we know how to determine if a graph has an Euler circuit, but if it does, how do we find one? While it usually is possible to find an Euler circuit just by pulling out your pencil and trying to find one, the more formal method is Fleury's algorithm.

Fleury's algorithm for finding an Euler Circuit or Path

1. Determine the number of odd vertices in the graph. (If the graph is not connected, there is no Euler Circuit or Euler Path.)

if no odd vertices it has EC	2 odd vertices it has EP	otherwise no EC or EP

2. Select a starting vertex.

If we have an EC, start at any vertex

If we have an EP start @ one of the odd vertex & end @ the other one.

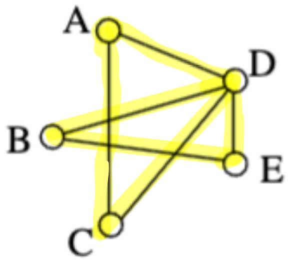
3. At each step, choose any edge leaving your current vertex but if you have a choice, don't choose a bridge of the yet-to-be-traveled part of the graph. → recall

4. Add that edge to your circuit, and delete it (mark it) from the graph.

5. Continue until you're done. (covered all the edges).

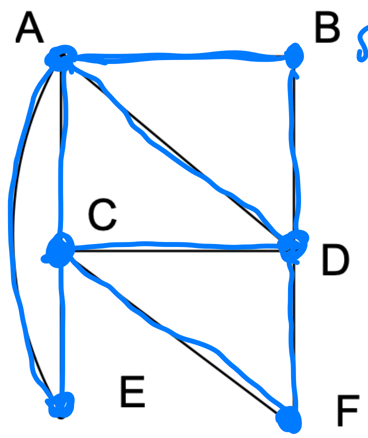
Example 6. Find an Euler Circuit in the following graph using Fleury's Algorithm.

EC



1) connected ✓ & no odd vertices
2) any vertex as start:
ADEBDCA → Euler circuit

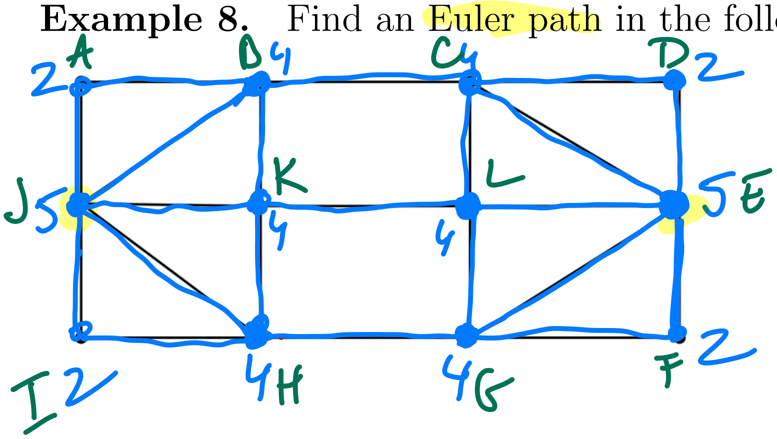
Example 7. Find an Euler Circuit in the following graph using Fleury's Algorithm.



start

1) connected (all even degree)
2) start at B
BADCEACFDB

Example 8. Find an Euler path in the following graph using Fleury's Algorithm. ✓



1) connected (2 odd vertices)
2) start @ E or J

EFGEDCELCBAJBKJIHGLKHJ