Math 142: Monday, September 30th, Day 13 Notes

Agenda for Monday, Sep 30th 2024

Coming up

- ! HW 8 due today, Sept 30th
- ! HW 9 due Wednesday, Oct 2nd
- ! Second **Excel Homework** due Wednesday, Oct 2nd
- Student Hours Tuesday, 1-2:15pm

Recall: experiment ex: tossing a coin

outcome ex: H, T

sample space ex: $S = \{H, T\}$

$n(S) = 2$

event ex: A = the event of the coin landing on T.

$A = \{T\}$

classical prob (outcomes were equally likely to happen) ex:

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

from previous notes

Example 4. Classical probability Experiment: Rolling a fair 12 sided die.

S = {1, 2, 3, . . . , 11, 12} n(S) = 12

Event A: rolling a prime number A = {2, 3, 5, 7, 11} n(A) = 5

Event B: rolling a number divisible by 5 B = {5, 10} n(B) = 2

Event C: rolling a number greater than 4 which is divisible by 3

C = {6, 9, 12} n(C) = 3

Find the following probabilities.

$P(A) = \frac{n(A)}{n(S)} = 5/12 = 0.417$ or 41.7%.

$P(B) = \frac{2}{12} = \frac{1}{6} = 0.1666\ldots = 0.167$ 16.7%.

$P(C) = \frac{3}{12} = \frac{1}{4} = 0.250$

Empirical (Experimental) Probability - used when outcomes are not equally likely.

$P(A) = \frac{\text{number of times A happens}}{\text{number of total trials}} = \frac{\text{frequency}}{\text{sample size}}$

Example 5. Julia made a die for a game. It had 5 faces, labeled 1-5. Because of the shape of the die, it was biased in favor of rolling a 3. She experimented to estimate how likely it was for a 3 to be rolled. Julia rolled the die 100 times, and the number of times she rolled a 3 was 45.

$P(3) = \frac{45}{100} = 0.45$

Example 6. Empirical Probability: Choosing a student from a survey.

	Accounting	Non Accounting	
Freshman	20	50	70
Soph	14	56	70
	34	106	140

$n = 140$ sample size

1. What is the probability of choosing a student who is majoring in accounting?

$$P(\text{Acct}) = \frac{\text{frequency}}{\text{sample size}} = \frac{34}{140} = 0.2428 = 0.243$$

2. What is the probability of choosing a student who is a sophmore?

$$P(\text{Soph}) = \frac{70}{140} = \frac{1}{2} = 0.5$$

3. What is the probability of choosing a student who is not majoring in accounting?

$$P(\text{not Acct}) = \frac{106}{140} = 0.757$$

$$P(\text{not Acct}) = \frac{140 - 34}{140} = \frac{140}{140} - \frac{34}{140} = 1 - P(\text{Acct})$$

Properties of Probability

1. For any event A, $\mathbf{0 \le P(A) \le 1}$. That is, the probability of any event is always a number between 0 and 1.
2. The probability of any event A is equal to the sum of the probabilities of the individual outcomes in A.
3. The sum of the probabilities of all outcomes in S must equal 1. This is true whether S consists of equally likely outcomes.

(2) $S = \{1,2,3,4,5,6\}$ rolling a die

$A = \{2,4,6\}$ $A \to$ even number

$$P(\text{even number}) = P(2) + P(4) + P(6)$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

(3) ✓

Often we deal with events that are described in terms of other events. These are called compound events and they involve the connectors *or*, *and* and *not*.

Example 1. Consider the experiment of rolling a six sided die and flipping a coin.

Sample space: $S = \{1T, 1H, 2T, 2H, 3T, 3H, 4T, 4H, 5T, 5H, 6T, 6H\}$.

Let event A be the event of rolling an even number and event B be the event of getting Tail on the coin.

$A = \{2H, 2T, 4H, 4T, 6H, 6T\}$

$B = \{1T, 2T, 3T, 4T, 5T, 6T\}$

The event A OR B includes outcomes that are in A or in B or in Both without writing duplicates

A or B = $\{2H, 1T, 2T, 4H, 4T, 3T, 5T, 6H, 6T\}$

The event A AND B includes the outcomes in common to A and B.

A and B = $\{2T, 4T, 6T\}$

The Complement of event A is denoted with A' (read A prime)

- A' consists of all outcomes that are not in A

$S = \{1, 2, 3, 4, 5, 6\}$ rolling a die

$A = \{1, 3, 5\}$ odd numbers $A' = \{2, 4, 6\}$

Example 2. The sample space S is the set of the first 20 whole numbers. Let event A = "the even numbers" and event B = "numbers greater than 13". Fill in the blanks:

1. $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

   $B = \{14, 15, 16, 17, 18, 19, 20\}$

2. $P(A) = \frac{10}{20} = \frac{1}{2}$

   $P(B) = 7/20$

3. A and B = $\{14, 16, 18, 20\}$

4. A or B = $\{2, 4, 6, 8, 10, 12, 14, 15, 16, 17, 18, 19, 20\}$

5. P(A and B) = $\frac{4}{20}$

6. P(A or B) = $13/20$

7. A' = $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

   $P(A') = \frac{10}{20} = \frac{1}{2}$

8. $P(A) + P(A') = \frac{10}{20} + \frac{10}{20} = \frac{1}{2} + \frac{1}{2} = 1$

9. $P(A|B) =$

   $P(B|A) =$

next class