

Total Possible Points: 100+ 2 bonus points

Name: _____

1. A local pizzeria offers the following options for their build-your-own pizza: (3)
 2 types of crust (regular, deep dish), 4 types of sauce (tomato, alfredo, BBQ, pesto) and 5 types of cheese (mozzarella, cheddar, provolone, parmesan, gouda).

$$2 \cdot 4 \cdot 5 = 40$$

How many different pizzas can be made?

- A. 245 B. 40 C. 11 D. 82

2. Evaluate ${}_{10}C_5$ (3)

- A. 210 B. 5040 C. 252 D. 120 E. none of these

3. A high school musical has leading roles that must be assigned to 5 different students. If 8 students audition for these roles, how many different ways can the director assign the roles if each student can only have one role and the roles are distinct (meaning each of the student is assigned a specific role)? (3)

- A. 792 B. 56 C. 6,720 D. 336

$$8P5$$

4. A student council needs to select 4 representatives from a group of 18 candidates. If each position must be filled by a different person, how many possible selections are there? (3)

- A. 73,440 B. 18,564 C. 43,758 D. 24,024

$$18C4$$

5. A student takes an 8-question multiple choice quiz with 5 choices per question. If the student guesses randomly on each question, find: (6)

- 3 (a) The probability of getting exactly 3 questions correct: $n=8, p=\frac{1}{5}$

- A. 0.147 B. 0.186 C. 0.201 D. 0.215 E. none of the above

- 3 (b) The probability of getting all questions correct

$$0.00000256$$

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6. The table shows the distribution of daily coffee consumption (in cups) for adults in a city: (6)

Cups of Coffee	Probability
0	0.15
1	0.35
2	0.28
3	0.14
4	0.06
5	0.02

- 1 (a) Does this represent a probability distribution? **yes**
- 2 (b) Find $P(1 \leq X \leq 3)$ where X is the number of cups consumed.
 A. 0.63 **B. 0.77** C. 0.83 D. 0.92
 $0.35 + 0.28 + 0.14$

- 3 (c) What is the expected value $E(X)$?
 $0(0.15) + 1(0.35) + \dots + 5(0.02) = 1.67$

7. At a company, 40% of employees work remotely. If 15 employees are randomly selected: (10)

$$p = 0.4, n = 15$$

- 2 (a) Find $P(X = 6)$ where X is the number who work remotely.
 $\text{binomial pdf}(15, 0.4, 6) = 0.2065$
- 2 (b) Find $P(X \leq 8)$
 $\text{binomial cdf}(15, 0.4, 8) = 0.9049$
- 2 (c) Find $P(X \geq 5)$
 $1 - \text{binomial cdf}(15, 0.4, 4) = 0.78272$
- 2 (d) Find $P(X < 9)$
 $\text{binomial cdf}(15, 0.4, 8) = 0.9049$
- 2 (e) Find $E(X)$ $= np = 15(0.4) = 6$

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8. Let $U = \{1, 3, 5, 7, m, n, p, q, r\}$. (3)

Find the complement of the set $P = \{3, 7, n, q\}$.

A. $\{1, 2, 5, m, p, r\}$

B. $\{1, 5, m, p, r\}$

C. $\{1, 5, m, r\}$

D. $\{1, 3, 5, 7, m, n, p, q, r\}$

9. Let $U = \{k, l, m, n, o, p, q, r, s, t\}$ (5)

Let $A = \{k, m, o, q, s\}$

Let $B = \{k, m, s, t\}$

Let $C = \{p, q, r, s, t\}$

Let $D = \{r, t\}$

- 2 (a) Find $(A \cup B) = \{k, m, o, s, t, q\}$
1 (b) Find $(A \cap D) = \emptyset$
1 (c) Find $(A - B) = \{o, q\}$
1 (d) Find $|B| = 4$

10. List the elements in the set $\{x \mid x \text{ is an odd natural number less than } 10\}$ (3)

A. $\{1, 3, 5, 7, 9\}$

B. $\{3, 5, 7\}$

C. $\{1, 3, 5, 7\}$

D. $\{1, 2, 3, 4, 5\}$

11. Complete each blank with either $\in$, $\notin$, $\subseteq$, or $\not\subseteq$ to make the statement true. (4)

1 (a) $6 \in \{3, 4, 5, 6, 7\}$

1 (b) $8 \notin \{3, 4, 5, 6, 7\}$

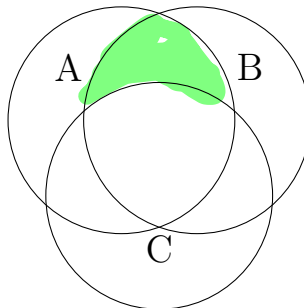
2 (c) $\{3, 7\} \subseteq \{3, 4, 5, 6, 7\}$

+1 (d) Bonus: $\emptyset \subseteq \{3, 4, 5, 6, 7\}$

+1 (e) Bonus: $\emptyset \in \{\emptyset\}$

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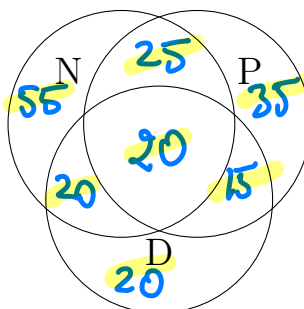
12. Shade the region representing $(A \cap B) - C$ in the Venn diagram below: (5)



13. In a survey of 250 people about their streaming service subscriptions: (10)

- 120 subscribe to Netflix
- 95 subscribe to Prime Video
- 75 subscribe to Disney+
- 45 subscribe to both Netflix and Prime Video
- 35 subscribe to both Prime Video and Disney+
- 40 subscribe to both Netflix and Disney+
- 20 subscribe to all three

Use the Venn diagram below to help solve this problem:



- 4 (a) How many people subscribe to exactly one of these services? $55 + 35 + 20 = 110$
- 3 (b) How many people subscribe to Netflix only? 55
- 3 (c) How many people subscribe to no subscriptions? $250 - (190) = 60$

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14. Determine whether the following statement is true or false: (4)

2 (a) $-2 \in \mathbb{N}$

A. True

B. False

2 (b) $\mathbb{Q} \subseteq \mathbb{R}$

A. True

B. False

15. Write the negation of the statement: "All mammals are cats." (4)

A. Not all mammals are cats.

B. Some mammals are not cats.

C. Some cats are mammals.

D. All mammals are cats.

16. Write the negation of the statement: "Some students are not seniors." (4)

A. All students are not seniors.

B. All students are seniors.

C. No seniors are students.

D. Some students are seniors.

17. Construct a truth table for the statement: $\sim p \wedge \sim q$. Add any columns you might find helpful. (6)

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

4

Give an example of an equivalent statement to $\sim p \wedge \sim q$.

2 $p \vee q$

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18. Construct a truth table for the statement: $\sim (p \vee q) \wedge \sim p$. Add any columns you might find helpful. (6)

p	q	$p \vee q$	$\sim (p \vee q)$	$\sim (p \vee q) \wedge \sim p$	$\sim p$
T	T	T	F	F	F
T	F	T	F	F	F
F	T	T	F	F	T
F	F	F	T	T	T

19. Construct the following truth table. (12)

3 points

p	q	r	$\sim p$	$\sim q$	$\sim r$	$\sim p \wedge p$	$\sim p \vee \sim q$	$\sim r \wedge q$	$(\sim p \vee \sim q) \vee (\sim r \wedge q)$
T	T	T	F	F	F	F	F	F	F
T	T	F	F	F	T	F	F	T	T
T	F	T	F	T	F	F	T	F	T
T	F	F	F	T	T	F	T	F	T
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	T	F	T	T	T
F	F	T	T	T	F	F	T	F	T
F	F	F	T	T	T	F	T	F	T

Identify any **tautologies** or **contradictions** in the table.

↓
1 $\sim p \wedge p$

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