MTH 122 — Formulas to Memorize

Days 1–4: Sections 9.1–9.4 and 2.1

9.1 Sequences and Their Notation

- **Sequence notation:** $\{a_n\} = \{a_1, a_2, a_3, \ldots, a_n, \ldots\}$, where a_n is the nth (general) term and n is a positive integer.
- **Alternating sequences:** include a factor of

$$(-1)^n \quad (\text{signs } -, +, -, +, \ldots) \qquad \text{or} \qquad (-1)^{n+1} \quad (\text{signs } +, -, +, -, \ldots)$$

- **Fibonacci sequence (recursive):**

$$a_1 = 1, \quad a_2 = 1, \quad a_n = a_{n-1} + a_{n-2}$$

- **Factorial:**

$$n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n, \qquad 0! = 1, \qquad n! = n\,(n-1)!$$

9.2 Arithmetic Sequences

- **Form:** $a,\ a+d,\ a+2d,\ a+3d,\ \ldots$ (a = first term, d = common difference)
- **Common difference:** $d = a_{n+1} - a_n$
- **nth term:**

$$a_n = a + (n-1)d$$

- **Recursive formula:**

$$a_1 = a, \qquad a_n = a_{n-1} + d$$

9.3 Geometric Sequences

- **Form:** $a,\ ar,\ ar^2,\ ar^3,\ \ldots$ (a = first term, $r \neq 0$ = common ratio)
- **Common ratio:** $r = \dfrac{a_{n+1}}{a_n}$
- **nth term:**

$$a_n = a\,r^{\,n-1}$$

9.4 Series and Sigma Notation

- **nth partial sum (sigma notation):**

$$S_n = a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$$

- **Partial sum of an arithmetic sequence:**

$$S_n = \frac{n}{2}\big[2a + (n-1)d\big] \qquad \text{or} \qquad S_n = n\left(\frac{a + a_n}{2}\right)$$

- **Partial sum of a geometric sequence** ($r \neq 1$):

$$S_n = a\,\frac{1 - r^n}{1 - r}$$

- **Sum of an infinite geometric series:**

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1 - r} \quad \text{if } |r| < 1 \text{ (i.e. } -1 < r < 1)$$

  If $|r| \geq 1$, the series **diverges**.

2.1 The Coordinate Plane and Circles

- **Quadrant signs:**

$$\text{I: } (+,+) \qquad \text{II: } (-,+) \qquad \text{III: } (-,-) \qquad \text{IV: } (+,-)$$

- **Distance formula:**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- **Midpoint formula:**

$$M = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$$

- **Pythagorean theorem** (checking for a right triangle, c = longest side):

$$a^2 + b^2 = c^2$$

- **Standard form of a circle** (center (h, k), radius r):

$$(x - h)^2 + (y - k)^2 = r^2$$

  Centered at the origin: $x^2 + y^2 = r^2$.

- **General form of a circle:**

$$x^2 + y^2 + Ax + By + C = 0$$

- **Completing the square:** for $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$ to both sides:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

- **Diameter endpoints given:** center = midpoint of the endpoints; radius = distance from center to an endpoint (half the diameter).