1. Rational Functions

In this section we will review rational functions and concentrate on how to graph them by hand.

A rational function is a function of the form $f(x) = \dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials. Even though rational fractions are constructed from polynomials, their graphs look quite different from the graphs of polynomial functions.

Examples of rational functions:
For each function, decide if it is a rational function and then if it is find its domain, x-intercepts and y-intercept.

1. $f(x) = \dfrac{x^2+3}{(x-1)(x+2)}$.

2. $f(x) = \dfrac{x^2-3}{(x^2+1)(x+2)}$.

3. $f(x) = \dfrac{x^2+3}{(\sqrt{x}-1)(x+2)}$.

The **domain of a rational function** consists of all real numbers x except those for which the denominator is zero. When graphing a rational function, we must pay special attention to the behavior of the graph near those x-values.

2. A simple rational Function $f(x) = \dfrac{1}{x}$

Graph the rational function $f(x) = \dfrac{1}{x}$ and state its domain and range.

Begin by factoring the numerator and the denominator. If both the denominator and the numerator have a common factor that reduces, then we say that the rational function has a **hole** at the corresponding x of that factor.

Vertical Asymptotes - A rational function has vertical asymptotes where the denominator is zero (after any cancellation of common factors from the denominator and numerator)

Horizontal asymptotes

Case 1: If the degree of the numerator is less than the degree of the denominator, the the horizontal asymptote is $y = 0$.

Case 2: If the degree of the numerator equals the degree of the denominator, then the horizontal asymptote is $y =$ the ratio of the leading coefficients.

Case 3: Lastly, if the degree of the numerator is higher than the degree of the denominator, then the rational function has no horizontal asymptotes.

We have one more type of asymptote for rational functions.

Let $f(x) = \dfrac{P(x)}{Q(x)}$ be a rational function. If the degree of the top polynomial $P(x)$ is exactly one more than the degree of the bottom polynomial $Q(x)$, then the function $f(x)$ will have a slant or oblique asymptote of the form $y = mx + b$.

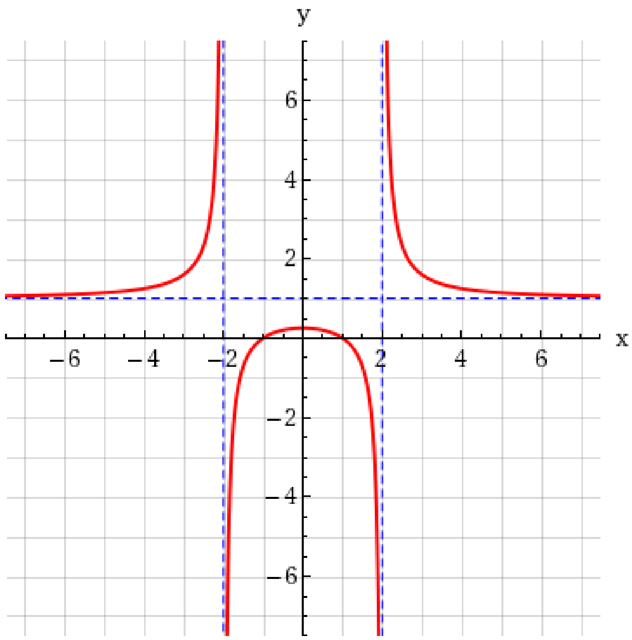
Example: How to get the slant asymptote. Let $f(x) = \dfrac{x^3}{2x^2 - 2}$.

Find the asymptotes and holes(if any) of the rational function

$$f(x) = \frac{(x^2-1)(2x+3)}{(x+1)(x^2+5x+6)}$$

4. Graphs of Rational functions

The graphs of rational functions are more complicated due to the asymptotes. Lets examine the graph of the rational function below.



How to graph Rational functions

Graph the rational function $f(x) = \dfrac{x^2 - 2x - 15}{x^2 + 4x}$.

Practice: Graph the rational function $f(x) = \dfrac{x^2 - 4x - 12}{x^2 - x - 6}$