Day 24 Notes

7.3 Systems of Nonlinear Equations

1. Introduction to Systems of Nonlinear Equations

A **system of nonlinear equations** is a system of two or more equations in two or more variables containing at least one equation that is not linear.

Recall that a linear equation can take the form $Ax + By + C = 0$. Any equation that cannot be written in this form is nonlinear.

The methods for solving systems of nonlinear equations are similar to those for linear systems:

- Substitution Method
- Elimination Method

2. Solving Using Substitution

The substitution method involves:

1. Solve one equation for one variable
2. Substitute the result into the second equation
3. Solve for the remaining variable
4. Check your solutions in both equations

Intersection of a Parabola and a Line

There are three possible types of solutions for a system involving a parabola and a line:

- **No solution:**
- **One solution:**
- **Two solutions:**

Example 1: Solve the system of equations.

$$\begin{aligned} x - y &= -1 \\ y &= x^2 + 1 \end{aligned}$$

Intersection of a Circle and a Line

Just as with a parabola and a line, there are three possible outcomes when solving a system of equations representing a circle and a line:

- **No solution:**

- **One solution:**

- **Two solutions:**

Example 2: Find the intersection of the given circle and line by substitution.

$$x^2 + y^2 = 25$$
$$x - y = 1$$

3. Solving Using Elimination

When both equations in the system have like variables of the second degree, solving them using elimination by addition is often easier than substitution.

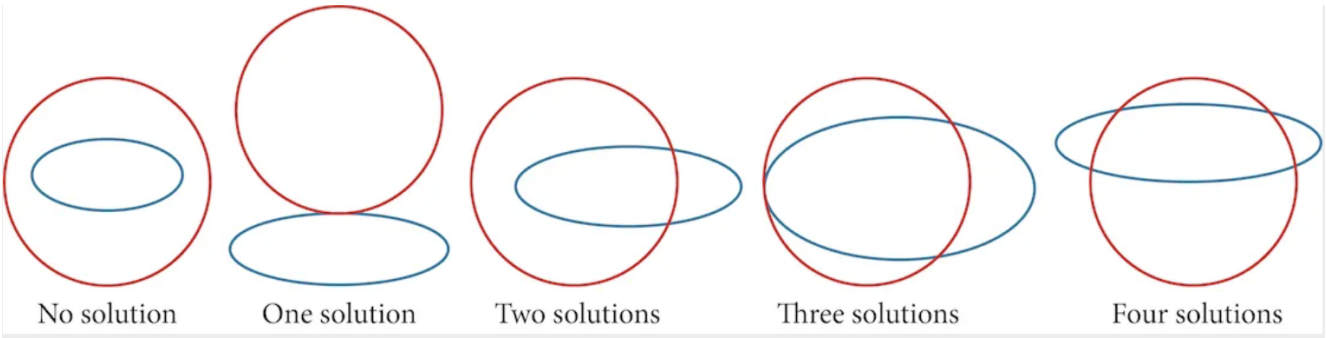
The elimination method is particularly useful for systems involving:

- Two circles
- A circle and an ellipse
- Two ellipses

Intersection of a Circle and an Ellipse

There are five possible types of solutions when solving a system of equations representing a circle and an ellipse:

- **No solution:**
- **One solution:**
- **Two solutions:**
- **Three solutions:**
- **Four solutions:**



Example 3: Solve the system of nonlinear equations.

$$x^2 + y^2 = 26$$
$$3x^2 + 25y^2 = 100$$

Practice Problems

Solve each system of nonlinear equations using substitution or elimination.

Problem 1:

$$\begin{aligned} y &= x + 1 \\ x^2 + y^2 &= 25 \end{aligned}$$

Problem 2:

$$\begin{aligned} y &= -x \\ x^2 + y^2 &= 9 \end{aligned}$$

Problem 3:

$$x^2 + y^2 = 25$$
$$x^2 - y^2 = 1$$

Problem 4:

$$2x^2 + 4y^2 = 4$$
$$x^2 + y^2 = 4$$