Using Sum and Difference Formulas

The sum and difference formulas allow us to express the sine, cosine, and tangent of a sum or difference of angles in terms of the sine, cosine, and tangent of the individual angles. These formulas are particularly useful for evaluating trigonometric expressions, solving equations, and proving identities.

1) $\sin(u+v) = \sin u \cos v + \cos u \sin v$

2) $\sin(u-v) = \sin u \cos v - \cos u \sin v$

3) $\cos(u+v) = \cos u \cos v - \sin u \sin v$

4) $\cos(u-v) = \cos u \cos v + \sin u \sin v$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Example 1 **Evaluating a Trigonometric Function**

Find the exact value of

1. $\cos 75° = \cos(30° + 45°)$

$= \cos 30° \cos 45° - \sin 30° \sin 45°$

$= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{1}{2} \frac{\sqrt{2}}{2}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

2. $\sin \frac{\pi}{12}$. $= \sin\left(\overset{u}{\frac{\pi}{3}} - \overset{v}{\frac{\pi}{4}}\right)$

(2)

$= \sin \frac{\pi}{3} \cdot \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \cdot \sin \frac{\pi}{4}$

$= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

Example 2 Evaluating a Trigonometric Expression

Find the exact value of $\sin(u+v)$ given $\sin u = 4/5$, where $0 < u < \pi/2$, and $\cos v = -12/13$, where $\pi/2 < v < \pi$.

Solution:

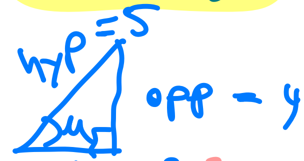
$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$= \frac{4}{5}\left(\frac{-12}{13}\right) + \cos u \sin v$$

$$= \frac{4}{5}\left(\frac{-12}{13}\right) + \frac{3}{5}\cdot\frac{5}{13}$$

$$= \frac{-33}{65}$$

$$\sin u = \frac{4}{5}$$



$$\cos u = \frac{3}{5}$$

$$\sin v = ?$$

$$\cos v = -\frac{12}{13}$$

$$\sin^2 v + \cos^2 v = 1$$

$$\sin^2 v + \left(\frac{-12}{13}\right)^2 = 1$$

$$\sin^2 v = 1 - \frac{144}{169}$$

$$\sin^2 v = \frac{25}{169}$$

$$\sin v = \pm\frac{5}{13}$$

Example 4 Proving a Cofunction Identity

Prove the cofunction identity $\cos\left(\frac{\pi}{2}-x\right)=\sin x$.

Solution:

$$\cos\left(\frac{\pi}{2}-x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\cdot\sin x$$

$$= 0\cdot\cos x + 1\cdot\sin x$$

$$= 0 + \sin x$$

$$= \sin x$$

Example 5 Solving a Trigonometric Equation

Find all solutions of

$$\sin\left(x+\frac{\pi}{4}\right)+\sin\left(x-\frac{\pi}{4}\right)=-1$$

in the interval $[0,2\pi)$.

Solution:

$$\sin x\cos\frac{\pi}{4} + \cancel{\sin\frac{\pi}{4}\cos x} + \sin x\cos\frac{\pi}{4} - \cancel{\sin\frac{\pi}{4}\cos x} = -1$$

$$2\sin x\cdot\cos\frac{\pi}{4} = -1$$

$$\frac{\cancel{2}\sin x\cdot\frac{\sqrt{2}}{\cancel{2}}}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$\sin x = -\frac{1}{\sqrt{2}} \Rightarrow \sin x = \frac{-\sqrt{2}}{2}$$

$$x = \frac{5\pi}{4}$$

$$x = \frac{7\pi}{4}$$