Thursday, Aprie 24th

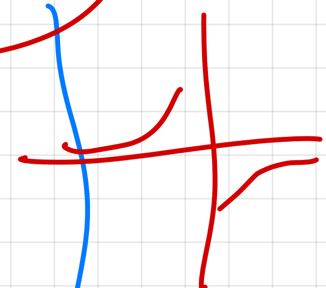
One more example w/ find inverses:

Ex Find the inverse of

$$f(x) = \frac{5}{x+2}$$

Range of f : Domain of f^{-1}

H.A. : $y = 0$



1) Step 1 Write $y = \frac{5}{x+2}$

Step 2 Switch x and y :

$$(y+2)\ \boxed{x = \frac{5}{\cancel{y+2}}}\ \cancel{(y+2)}$$

Step 3 Solve for y :

$$x(y+2) = 5$$

$$xy + 2x = 5$$

$$\frac{xy}{x} = \frac{5-2x}{x}$$

$$y = \frac{5-2x}{x}$$

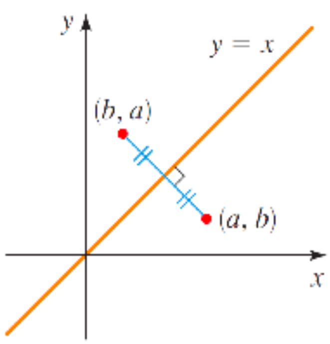
$$\boxed{f^{-1}(x) = \frac{5-2x}{x}}$$

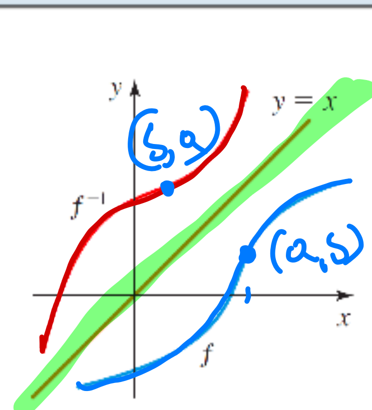
$x \neq 0$

4. How to find the inverse of a one-to-one function - Graphically

Let f be a one-to one function. Then $f(x) = y$ for any x in the domain of f and f has an inverse f^{-1}. A point (a, b) on the graph of f is going to become (b, a) on the graph of f^{-1} since the input is switched with the output.

The graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.

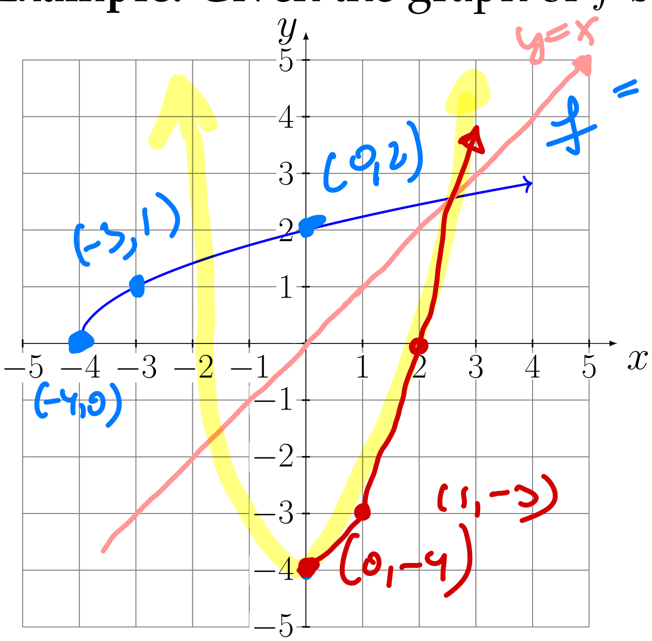




reflect f (blue) over y=x line to get the inverse function f^{-1}

Example: Given the graph of f below find the graph of f^{-1}.

$f = \sqrt{x+4}$



$f^{-1} = x^2 - 4 \quad , x \geq 0$

$(f \circ f^{-1})(x) = x$

$(f^{-1} \circ f)(x) = x$

$(f \circ f^{-1})(x) = f(f^{-1}(x))$

$= f(x^2-4)$

$= \sqrt{(x^2-4)+4} = \sqrt{x^2-4+4}$

$= \sqrt{x^2} = x$

1. Section 4.2 Exponential Functions

Exponential and logarithmic functions represent two fundamental mathematical relationships that are inverses of each other. Exponential functions, written in the form $f(x) = b^x$ where $b > 0$ and $b \neq 1$, model growth and decay phenomena where a quantity changes at a rate proportional to its current value. These functions appear in numerous real-world applications, from compound interest and population growth to radioactive decay.

An exponential function is a function of the form $f(x) =$ b^x, where the base b >0 and b $\neq 1$ $1^x = 1$
Note: The independent variable x is in the exponential !

Ex: $f(x) = 2^x$ $b = 2$

$f(x) = \left(\frac{1}{3}\right)^x$ $b = \frac{1}{3}$

$\to$ Euler's constant

The exponential function with base e is $f(x) =$ e^x, where $e = \left(1 + \frac{1}{n}\right)^n$ as $n \to \infty$. Note $e \approx$ 2.718...

Calculator buttons: e is on ÷ button

Recall rules of exponents:

$a^m \cdot a^n = a^{m+n}$ $(a^m)^n = a^{mn}$ $a^0 = 1$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\frac{a^m}{a^n} = a^{m-n}$ $(ab)^n = a^n b^n$ $a^{-n} = \frac{1}{a^n}$! $a^{\frac{1}{n}} = \sqrt[n]{a}$

$\frac{x^3}{x^1} = x^{3-1} = x^2$ $3^{-1} = \frac{1}{3^1} = \frac{1}{3}$

Example 1. Use a calculator(if needed) to evaluate the function at the indicated value of x. Round to three decimal places.

a. $f(x) = 5^x$ at $x = 2$

$f(2) = 5^2 = 25$

b. $f(x) = 2^x$ at $x = -3$

$f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

c. $f(x) = \left(\frac{1}{4}\right)^x$ at $x = 0$

$f(0) = \left(\frac{1}{4}\right)^0 = 1$

d. $f(x) = 2.3^x$ at $x = -1$

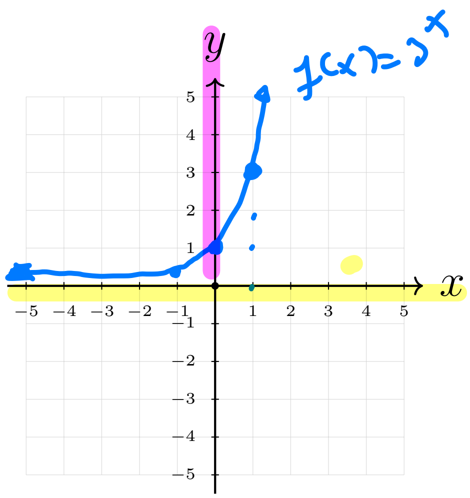
$f(-1) = (2.3)^{-1} = 0.435$

Example 2. Graph the function and write the domain and range in interval notation.

1) $f(x) = 3^x$

$b > 1$

x	$y = 3^x$
-1	$3^{-1} = \frac{1}{3}$
0	$3^0 = 1$
1	$3^1 = 3$



H.A. $\Rightarrow y = 0$

domain: $(-\infty, \infty)$

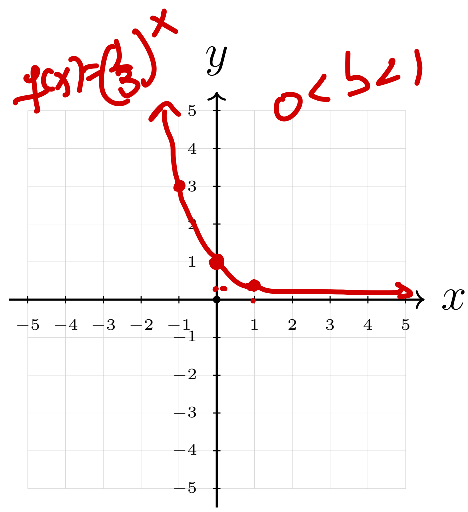
range $(0, \infty)$

increasing

exponential growth

2) $f(x) = \left(\frac{1}{3}\right)^x$

x	$y = \left(\frac{1}{3}\right)^x$
-1	$(1/3)^{-1} = 3$
0	$(1/3)^0 = 1$
1	$(1/3)^1 = 1/3$



H.A. $\Rightarrow y = 0$

domain $(-\infty, \infty)$

range : $(0, \infty)$

decreasing

exponential decay

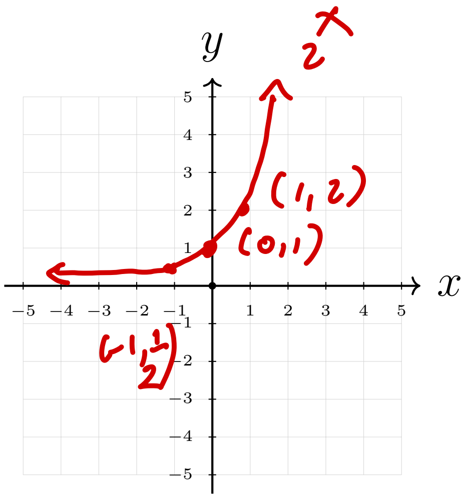
$3^x = 0$

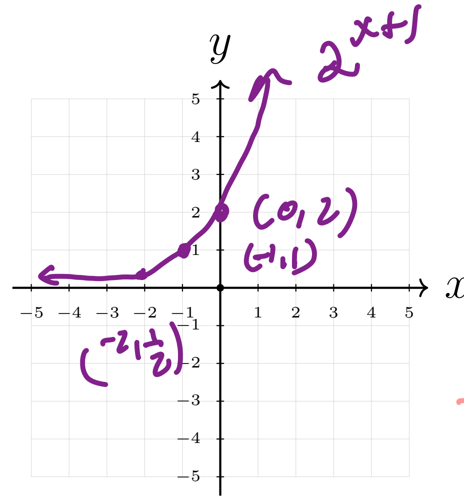
$3^? = 0$

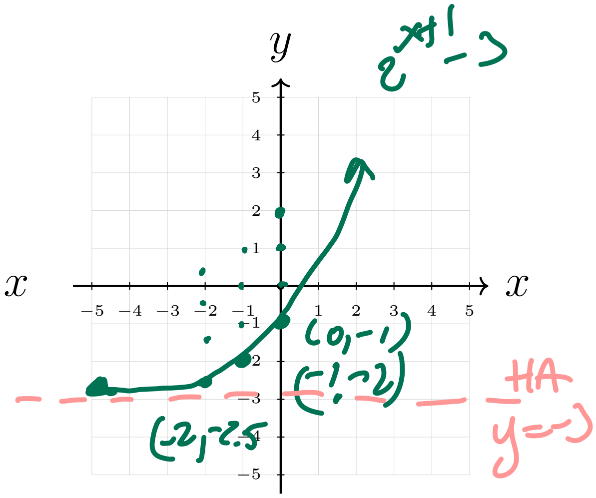
Example 3. Use the graph of the parent function to describe the transformation that yields the graph of g.

$g(x) = 2^{x+1} - 3$

$(x+1)^2 - 5$







2^x

HA: $y = 0$

2^{x+1}

left 1

HA: $y = 0$

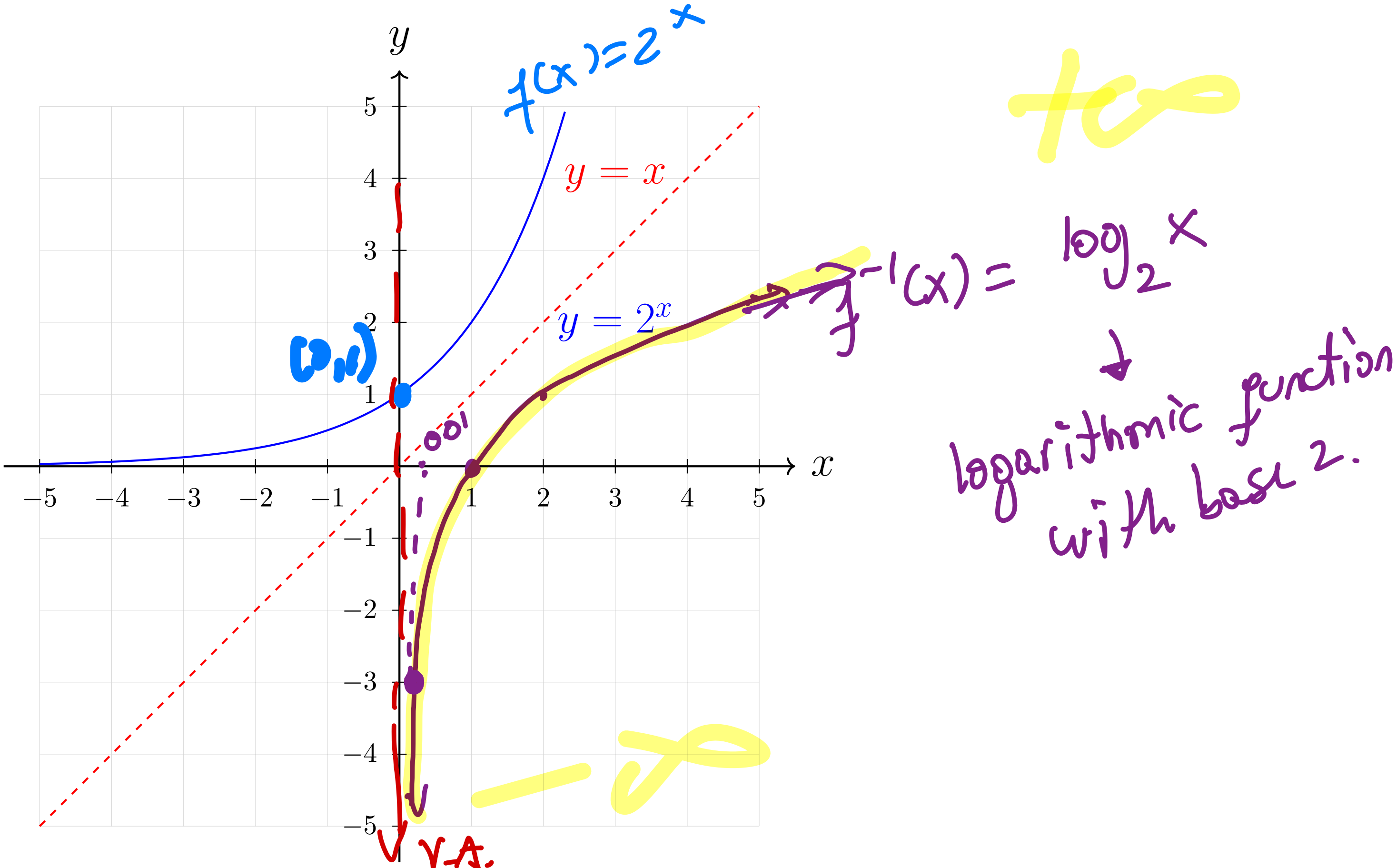
$2^{x+1} - 3$

down 3

$\frac{1}{2} - 3 = \frac{1}{2} - \frac{6}{2} = \frac{-5}{2}$
$= -2.5$

HA: $y = -3$

The inverse of the exponential function $f(x) = b^x$ is called **logarithmic function with base** b.



Notation for logarithmic function: $f(x) = \log_2 x$

Domain: of $\log_2 x$ is $(0, \infty)$ = range of 2^x

Range: of $\log_2 x$ is $(-\infty, \infty)$ = domain of 2^x

Asymptote: V.A. $x=0$

A logarithmic function is a function of the form $f(x) = \underline{\log_b x}$, where $x \underline{>0}$, and the base $b \underline{>0}$ and $b \underline{\neq 1}$.

I can't take log of a negative quantity!

Special Logarithms:

1) common log: $b=10$ $f(x)=\log_{10} x=\log x$

2) natural log: $b=e$ $f(x)=\log_e x=\ln x$

Example 4. Write the logarithmic equation in exponential form.

1. $\log_2 8 = 3$
2. $\log_{10} 7 = x$
3. $\log_{1/2} y = 5$
4. $\ln 1 = 0$

1) $\log_2 8=3$ $\quad 2^{\boxed{3}}=\boxed{8}$

2) $\log_{10} 7=x$ $\quad 10^{x}=7$

3) $\log_{\frac{1}{2}} y=5$ $\quad \boxed{\tfrac{1}{2}}^{\boxed{5}}=\boxed{y}$

4) $\ln 1=0$ $\boxed{\log_e 1=0}$ $\quad e^{0}=1$

Example 5. Write the exponential form in logarithmic equation.

1. $5^x = 23$
2. $8^{-2} = \dfrac{1}{64}$
3. $b^n = 9$

1) $5^x=23$ $\quad \log_5 23=x$

2) $8^{-2}=\frac{1}{64}$ $\quad \log_8 \frac{1}{64}=-2$

3) $\log_b 9=n$

Example 6. Use the definition of logarithmic function to evaluate the function at the indicated value of x without using a calculator.

1. $f(x) = \log_5 x$ at $x = 25$
2. $f(x) = \log_2 x$ at $x = 8$
3. $f(x) = \log x$ at $x = 100,000$
4. $f(x) = \ln x$ at $x = e^9$
5. $f(x) = \log x$ at $x = 0.001$
6. $f(x) = \ln x$ at $x = \frac{1}{e^5}$

1) $\boxed{\log_5 25=?}$ $\quad \log_5 25=2$

$5^{?}=25$ $\quad 5^2=25$

2) $\log_2 8=?$ $\quad \log_2 8=3$

$2^{?}=8$

$2^3=8$

3) $\log 100,000=?=5$

$10^{?}=100000$

$10^5=100000$

4) $\ln e^9=?=9$

$e^{?}=e^9$

5) $\log 0.001=?=-3$

$10^{?}=0.001 \Rightarrow \boxed{10^{?}=10^{-3}}$

7. $f(x) = \log_4 x$ at $x = 1$

8. $f(x) = \log_2 x$ at $x = 0$

9. $f(x) = \log_2 x$ at $x = -16$

7) $\log_4 1 = 0$
8) $\log_2 0 =$ undefined
9) $\log_2 (-16) =$ undef.

$10^{2.09} = 123$

$10^{?} = 123$

Example 7. Use a calculator to evaluate the function at the indicated value of x.

1. $f(x) = \log x$ at $x = 123$
2. $f(x) = \ln x$ at $x = 2$

$f(123) = \log_{10} 123 = 2.09$

$f(2) = \ln 2 = 0.69$

Basic Properties of Logarithms with base b, where $b > 0$, $b \neq 1$, and $x > 0$

1) $\log_b 1 = 0$ because $b^{?} = 1$
2) $\log_b b = 1$ because $b^{?} = b$
3) $\cancel{\log_b b^x} = x$ because $f^{-1}(f(x)) = x$
4) $b^{\log_b x} = x$ because $f(f^{-1}(x)) = x$

EX Use the properties of logarithms to simplify the expression, without using a calculator.

1) $\cancel{\log_3 3}^{5x} = 5x$ P3

2) $\log_{\sqrt{2}} 1 = 0$ Prop 1

3) $6 \log_4(1/4) = 6 \underbrace{\log_4 4^{-1}}_{P3} = 6(-1) = -6$

4) $4^{\log_4(5x+y)} = 5x + y$ P4

$\frac{1}{4} = 4^{?}$