Math 140: Chapter 5 Logic

Statements

A statement is a declarative sentence that is either true or false but not both true and false at the same time. Each statement has a truth value. It is either True or False.
T F

Examples of statements:

1. Barack Obama was the president of the USA. T
2. The number 2 is even and less than 10. T
3. Jimmy Fallon was responsible for the development of graph theory. F

Examples that are not statements:

1. What is your name?
2. Pick that up!
3. This statement is false. This is an example of a paradox

Compound Statements

A compound statement is a statement that combines statements using logical connectors such as: not, and, or, if ... then, if and only if.

Examples of compound statements:

1. If you eat your vegetables, then you get a cookie.
2. Tom is taking MTH 140 or Tom is taking MTH 141.
3. I biked to work and I carried my lunch.

Not a compound statement: Phineas and Ferb TV show was funny.

Symbolic Logic

has the goal to translate statements into symbols, analyze or simplify them, and then translate them back into words.

We use lower case letters to denote simple statements.

Example:

p: $1+1=2$ (T)

r: It's raining outside (F).

Truth Tables

A truth table is a table that shows all possibilities for the truth values of a statement.

p	r
T	T
F	F
T	F
F	T

Negation

p its negation is $\sim p$, or $\neg p$

The negation of a true statement is __false__ and the negation of a false statement is __true__. (The statement and its negation make up all the possibilities).

Examples – Write the negation of each statement:

- p: My car is a Ford. F

$\sim p$: My car is not a Ford. T

- q: Today is not Thursday. T

$\sim q$: Today is Thursday. F

- r: $6x < 0$

$\sim r \quad 6x \geq 0$

- s: $y \geq 19$

$\sim s: \quad y < 19$

Quantifiers

universal quantifier

Words such as <u>all</u>, <u>some</u>, <u>none</u>, _______ are called quantifiers.

↳ existential quantifier.

Note: The word "some" in this context means <u>at</u> <u>least</u> <u>one</u>.

Examples of statements with quantifiers - Write the negation of each statement:

p: 1. All cars have airbags. F (example of a quantified statement)

~p: Some cars don't have airbags. (T)

p: 2. Some cars have antilock brakes. T

~p: No cars have antilock brakes. F

(All cars don't have antilock brakes)

p 3. No cars have window stickers. p: All cars don't have stickers.

~p: Some cars have window stickers.

p: 4. Some cars do not have air conditioning. T

~p: All cars have AC.

p: 5. At least one car has a bumper sticker. Some cars have bumper stickers.

~p: All cars don't have stickers.

or No cars have bumper stickers.

Summary for quantified statements

p	$\sim p$
all ... are (have)	some .. are not (don't have)
all ... are not (don't have)	some are (have).
some ... are (have)	all .. are not (don't have)
some .. are not (don't have)	all .. are (have)

Sets of numbers:

$\mathbb{N}$ = set of natural numbers = $\{1, 2, 3, \cdots\}$

W = whole numbers = $\{0, 1, 2, 3, \cdots\}$

$2 = \frac{2}{1}, 3 = \frac{3}{1}, \cdots$

$\mathbb{Z}$ = integers = $\{\ldots, -2, -1, 0, 1, 2, \cdots\}$

$\frac{1}{2}, -\frac{2}{3}, \cdots$

$\mathbb{Q}$ = rational numbers = $\{\frac{p}{q} \mid \underbrace{p, q \in \mathbb{Z}, q \neq 0}_{\text{such that } p, q \text{ are integers}}\}$

$\boxed{\text{Irrational numbers} = \mathbb{R} \setminus \mathbb{Q}}$

$\mathbb{R}$ = real numbers = $\{x \mid x$ can be written as a decimal$\}$ ex: $-1, 7, \frac{1}{3} = 0.3\overline{3}, \pi = 3.14\ldots$

True or False.

1. Every natural number is an integer. T
2. There exists an integer that is not a natural number. T
3. All irrational numbers are real numbers. T
4. Some whole numbers are not rational numbers. F
5. Each rational number is a positive number. F ex $-\frac{1}{2}$