Day 2 notes
9.2/9.3 Arithmetic and Geometric Sequences

1. Arithmetic Sequences

In this section we are going to introduce Arithmetic Sequences. The simplest way to generate an arithmetic sequence is to start with a number a and add to it a fixed constant d, over and over again.

positive or negative

Definition: An arithmetic sequence is a sequence of the form:

$a, a+d, a+2d, a+3d, \ldots, a+(n-1)d, \ldots$

$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$

The number a is <u>the first term</u> and d is called <u>the common difference</u>.

$a_4 - a_3 = a+3d - a - 2d = d$

The nth term of an arithmetic sequence is given by:

$$\boxed{a_n = a + (n-1)d}$$

Example 1: Is this sequence an arithmetic sequence? $13, 7, 1, -5, \cdots$.

a_1, a_2, a_3, a_4

$a_2 - a_1 = 7 - 13 = -6$, $a_3 - a_2 = 1 - 7 = -6$, $a_4 - a_3 = -5 - 1 = -6$

If yes, find the common difference, the next three terms, the nth term, and the 300th term of the arithmetic sequence

yes arithmetic w/ $d = -6$

$a_5 = -5 + (-6) = -11$

$a_6 = -11 + (-6) = -17$

$a_7 = -17 + (-6) = -23$

$a = 13$
$d = -6$

$a_5 = 13 + 4(-6)$
$= 13 - 24$
$= -11$

$$\boxed{a_n = a + (n-1)d} = \boxed{13 + (n-1)(-6)}$$

$a_{300} = a + (300-1)d$
$= 13 + (299)(-6) = -1781$

Example 2: Write the first five terms of the sequence $a_n = -4 + 3n$. Determine whether or not the sequence is arithmetic. If it is, find the common difference.

$\{a_n\} = \{-1, 2, 5, 8, 11, \ldots\}$

$+3 \; +3 \; +3 \; +3$

$a_n = -4 + 3n$

$a_1 = -4 + 3 \cdot 1 = -1$

$a_2 = -4 + 3(2) = 2$

$a_3 = -4 + 3(3) = 5$

$a_4 = 8$

$a_5 = 11$

yes $d = 3$

$a = -1$

Example 3: The 11th term of an arithmetic sequence is 52, and the 19th term is 92. Find the 1000th term.

$a_{11} = 52$ $a_{19} = 92$ $a_{1000} = ?$

$\boxed{a_n = a + (n-1)d}$

$a_{11} = a + (11-1)d = 52$

$a + 10d = 52$ *

$a_{19} = a + (19-1)d = 92$

$a + 18d = 92$ *

Solve $\begin{cases} a + 10d = 52 \\ a + 18d = 92 \end{cases}$ −

$\cancel{a} + 18d - \cancel{a} - 10d = 92 - 52$

$8d = 40 \Rightarrow d = 5$

Find a: $a + 10(5) = 52$

$a = 2$

$\boxed{a_n = 2 + (n-1)(5)}$

$a_{1000} = 2 + 999(5)$

$a_{1000} = 4997$

Arithmetic sequence recursive formula:

$$\boxed{\begin{aligned} a_1 &= a \\ a_n &= a + (n-1)d \end{aligned}}$$

Recursive formula:

$$\boxed{\begin{aligned} a_1 &= a \\ a_n &= a_{n-1} + d \end{aligned}}$$

*!

Next, we are going to introduce Geometric Sequences. A geometric sequence is generated when we start with a number a and repeatedly multiply by a fixed nonzero constant r.

Definition: A geometric sequence is a sequence of the form:

$$a, ar, ar^2, ar^3, \ldots\ ar^{n-1}, \ldots$$

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$$

The number a is the first ter and r is called common ratio. $\frac{a_2}{a_1} = \frac{ar}{a} = r$ or $\frac{a_4}{a_3} = \frac{ar^3}{ar^2} = r$

The nth term of a geometric sequence is given by:

$$\boxed{a_n = ar^{n-1}}$$

Example 5: If $a = 3$ and $\boxed{r = 2,}$ then we have the geometric sequence:

$\{a_n\}: \{3, 6, 12, 24, \ldots\}$

$a_n = ar^{n-1}$

$\boxed{a_n = 3 \cdot 2^{n-1}}$

$a_{10} = ?$

$a_{10} = 3 \cdot 2^{10-1} = 3 \cdot 2^9 = 1536$

Example 6: The sequence

$2, -10, 50, -250, 1250, \cdots$ is a geometric sequence with $a = 2$ and $r = -5$.
Find a_{10}.

$a_n = ar^{n-1}$

$a_n = 2(-5)^{n-1}$

$a_{10} = 2(-5)^{10-1} = 2(-5)^9$

$a_{10} = -3{,}906{,}250$

Example 6: Find the common ratio, the first term, the nth term, and the eighth term of the geometric sequence $5, 15, 45, 135, \cdots$

$a_1 \quad a_2 \quad a_3 \quad a_4$

$r = \frac{a_2}{a_1} = \frac{15}{5} = 3$

$a = 5$

$a_n = ar^{n-1} \Rightarrow \boxed{a_n = 5(3)^{n-1}}$

$a_8 = 5(3)^{8-1}$

$a_8 = 5 \cdot 3^7$

$= 10935$