MTH 122: Practice Final Exam

College of Lake County

Total Possible Points: 114

Name: _____

1. The n^{th} term of a sequence is given. Find the indicated term. Express your answer in simplest form. (3)

$$b_n = \frac{2n}{(n+3)!};\ \text{find } b_5.$$

2. Find the n^{th} term a_n of a sequence whose first four terms are given. (4)

$$-\frac{9}{4}, -\frac{10}{8}, -\frac{11}{12}, -\frac{12}{16}, \ldots$$

3. Find the sum. (3)

$$7 + 11 + 15 + \ldots + 71$$

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4. Find the sum of the geometric series, if possible. (4)

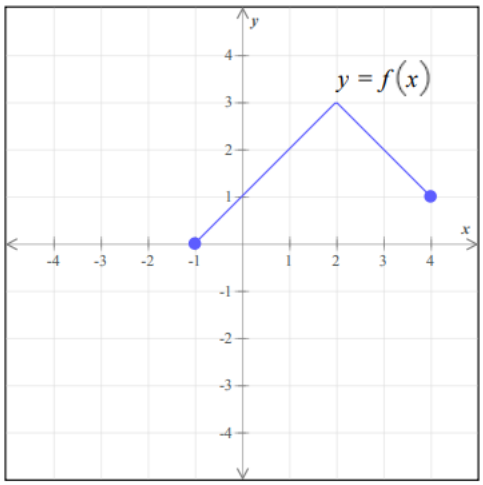
$$\sum_{k=1}^{\infty} \left(\frac{5}{6}\right)^{k-1}$$

A. The sum does not exist.

B. The sum does exist.

The sum is _____. Express the sum as an integer or simplified fraction.

5. The graph of $y = f(x)$ is given. Graph the function $y = f(-x) + 1$. (3)



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6. A lawn service company charges \$51 for each lawn maintenance call. The fixed monthly cost of \$657 includes telephone service and depreciation of equipment. The variable costs include labor, gasoline, and taxes and amount to \$36 per lawn. (6)

   (a) Write a linear cost function representing the monthly cost $C(x)$ for x maintenance calls.

   (b) Write a linear revenue function representing the monthly revenue $R(x)$ for x maintenance calls.

   (c) Write a linear profit function representing the monthly profit $P(x)$ for x maintenance calls.

   (d) Determine the number of lawn maintenance calls needed per month for the company to make money. Round up your answer to the nearest maintenance calls.

   (e) If 41 maintenance calls are made for a given month, how much money will the lawn service make or lose? Write your answer to the nearest cent.

7. Determine whether the function is even, odd, or neither. (3)

$$g(x) = \sqrt{4 + x^2}$$

   A. The function g is an even function.

   B. The function g is an odd function.

   C. The function g is neither even nor odd.

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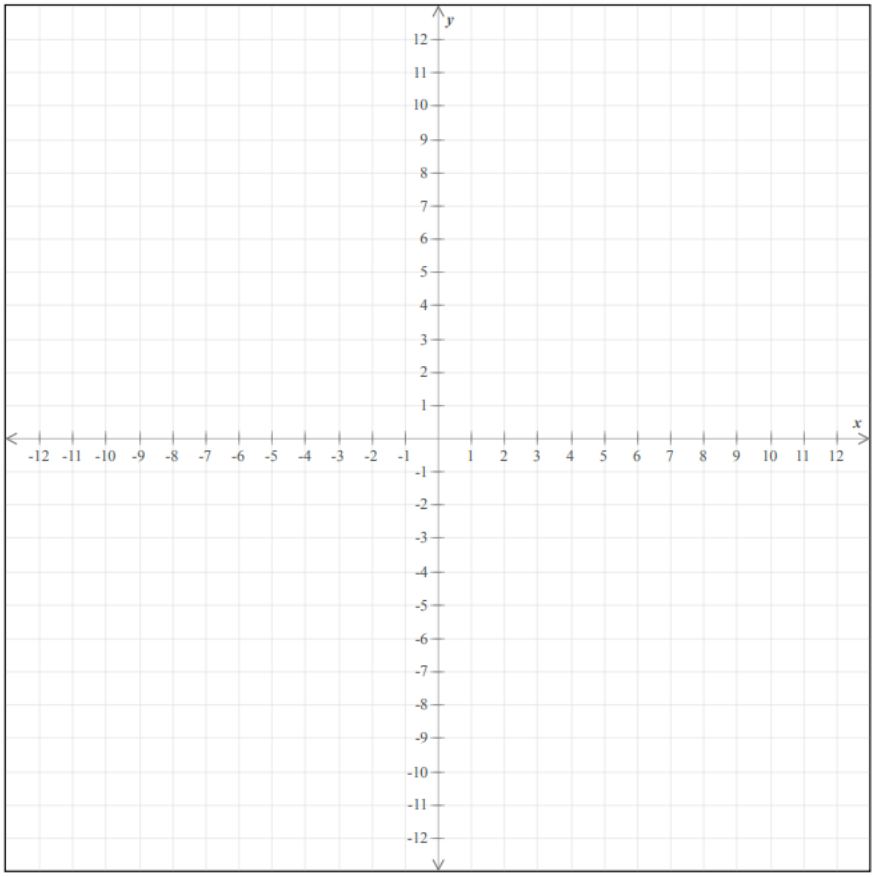
8. Consider the follwoing piecewise defined function. (8)

$$f(x) = \begin{cases} -x^2 + 4 & \text{for } x \leq 2 \\ 3x & \text{for } x > 2 \end{cases}$$

(a) Graph the function.



(b) Write the domain in the interval notation.

(c) Write the range in interval notation.

(d) Evaluate $f(-2)$, $f(2)$, and $f(3)$.

(e) Find the value(s) of x for which $f(x) = 9$.

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9. Given $f(x) = x^2 + 8x$, (5)

Find $\dfrac{f(x+h) - f(x)}{h}$ and simplify.

10. Given $k(x) = -5x + 2$ and $m(x) = \frac{1}{x}$, (4)

(a) Find $(k \circ m)(1)$.

(b) Find $(m \circ k)(2)$.

11. Find the coordinates of the vertex of the parabola by applying the vertex formula. (3)

$$f(x) = 2x^2 - 16x + 119$$

$(h, k) =$

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12. The gas mileage $m(x)$ (in mpg) for a certain vehicle can be approximated by $m(x) = -0.033x^2 + 2.637x - 35.013$, where x is the speed of the vehicle in mph. (4)

(a) Determine the speed at which the car gets its maximum mileage. Round your answer to the nearest mph.

(b) Determine the maximum gas mileage. Round your answer to one decimal place.

13. A polynomial $f(x)$ and three of its zeros are given. (5)

$$f(x) = 3x^5 + 17x^4 + 48x^3 + 24x^2 - 131x + 39;$$

$$-2 - 3i \text{ and } \frac{1}{3}, -3 \text{ are zeros}$$

(a) Is 1 a zero?

(b) Find all the zeros. Write the answer in exact form.

(c) Factor $f(x)$ as a product of linear factors.

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14. For the graph of (5)

$$f(x) = \frac{(3x+2)(x+6)}{(x+1)(3x+5)}$$

(a) Identify the x-intercepts.

(b) Identify any vertical asymptotes.

(c) Identify the horizontal asymptote or slant asymptote if applicable.

(d) Identify the y-intercepts.

15. A one-to-one function is given. Write an equation for the inverse function. (5)

$$t(x) = \frac{x-3}{x+4}$$

16. Write the logarithm as a sum or difference of logarithms. Simplify each term as much as possible. $\log\left(\frac{10{,}000}{\sqrt{p^2+q^2}}\right)$ (3)

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17. Write the logarithmic expression as a single logarithm with coefficient 1, and simplify as much as possible. Assume that the variables represent positive real numbers. (3)

$$5\log_7 m - 7\log_7 n - 6\log_7 p$$

18. Solve the equation. Write the solution set with the exact values given in terms of common or natural logarithms. Also give approximate solutions to at least 4 decimal places. (3)

$$6e^{3n-2} + 3 = 15$$

19. Solve the equation. Write the solution set with the exact solutions. Make sure to verify the solutions. (3)

$$\log_3(q) - 3 = -\log_3(q+6)$$

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20. Use the model $A = Pe^{rt}$ or $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the future value of P dollars invested at interest rate r compounded continuously or n times per year for t years. (5)

A \$10,000 investment grows to \$12,507.51 at 4.5% interest compounded quarterly. For how long was the money invested? Round to the nearest year.

21. A sample of radioactive material decays according to the model $Q = Q_0 e^{-kt}$, where Q is the amount of radioactive material (in grams) present at time t (in years), Q_0 is the initial amount, and k is the decay constant. The initial amount of the radioactive sample is 50 grams and it decays a rate of 3.5% per year. (8)

(a) How much of the original 50-gram sample will remain after 30 years? Express your answer in grams to the nearest hundredth.

(b) Find the half-life of this radioactive material. The half-life is the time required for half of any given amount of the material to decay.

(c) After how many years will only 5 grams of the original sample remain? Express your answer to the nearest year.

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22. Solve the system of equations by using any method. (4)

$$6x - 3y = -3$$
$$5x + 18y = -23$$

The solution set is _____.

23. Consider the follwoing matrices. (8)

$$A = \begin{bmatrix} 2 & 5 & 0 \\ -3 & 1 & 6 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 6 \\ 6 & 4 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

(a) What the the size of matrix A?

(b) What is the element a_{23}?

(c) Find $A + B$ if possible.

(d) Find $A - C$ if possible.

(e) Find AB if possible.

(f) Find $3C$ if possible.

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24. Write the augmented matrix for the given system. (2)

$$\begin{aligned} 6x + 4y - 6z &= 9 \\ 9x + 7z &= 3 \\ 2y - 9z &= 7 \end{aligned}$$

25. Determine if the matrix is in row-echelon form. If not, explain why. (2)

$$\begin{bmatrix} 1 & 2 & -4 & 7 \\ 0 & 1 & -5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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26. Solve the system by using Gaussian elimination or Gauss-Jordan elimination. (4)

$$\begin{aligned} 2x &= -71 - 9y - 4z \\ x + 5y &= 2z - 39 \\ 9z &= 17 - x + 3y \end{aligned}$$

The solution set is _____.

27. An augmented matrix is given. Determine the number of solutions to the corresponding system of equations. $\begin{bmatrix} 1 & 7 & 6 \\ 0 & 0 & 0 \end{bmatrix}$ (2)

A. The system has one solution.

B. The system has no solution.

C. The system has infinitely many solutions.

28. An augmented matrix is given. Determine the number of solutions to the corresponding system of equations. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ (2)

A. The system has one solution.

B. The system has no solution.

C. The system has infinitely many solutions.

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29. Are the following two 2×2 matrices are inverses of each other? Show work.

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{pmatrix}$$

30. Find the inverse of the following 2×2 matrix:

$$C = \begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}$$

31. Solve the following 3×3 system of linear equations using inverse matrices:

$$\begin{cases} 2x + y - z = 3 \\ x + 3y + 2z = 11 \\ 3x - y + z = 2 \end{cases}$$

32. Solve the following system of nonlinear equations.

$$\begin{cases} x^2 + y^2 = 25 \\ y = x + 1 \end{cases}$$