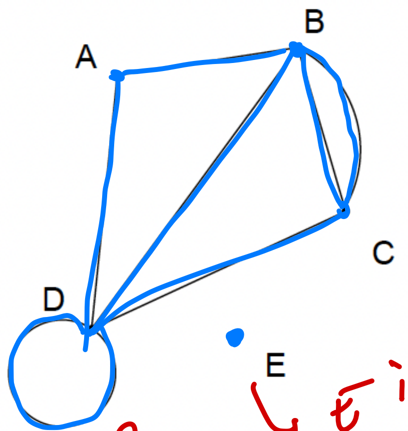
Math 140: Section 1.1 Intro to Graph Theory: Basic Concepts

Graph - A picture that contains a set of dots or circles, called **vertices**, and a set of **edges** that connect the vertices.

Example 1. Consider the graph below. Give the set of vertices and the set of edges:



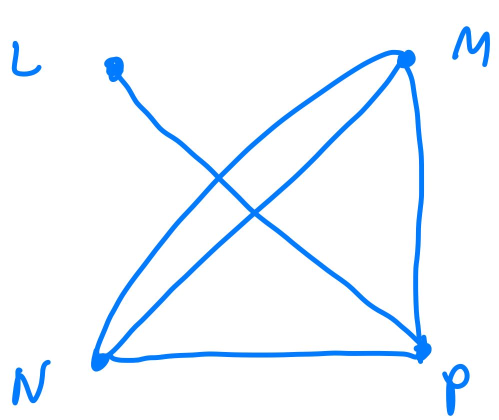
E is an example of isolated vertex

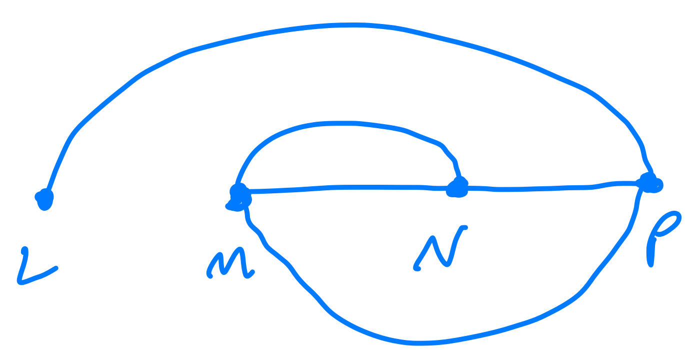
vertices: $V = \{A, B, C, D, E\}$

edges: $E = \{AB, BC, BC, DD, CD, AD, DD\}$

Example 2. Draw two different graphs for
$\mathcal{V} = \{L, M, N, P\}$ and $\mathcal{E} = \{LP, MM, PN, MN, PM\}$

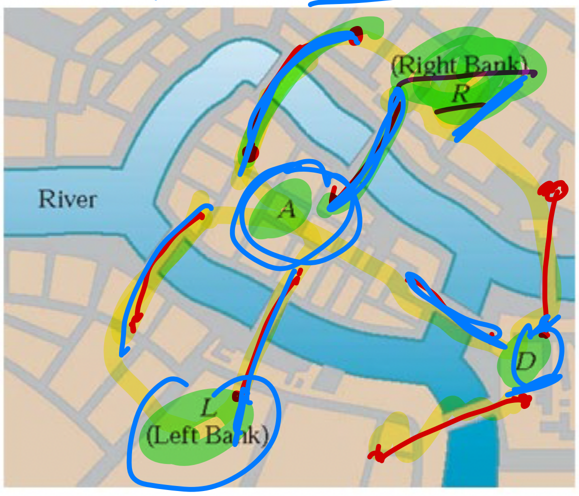
vertices   edes



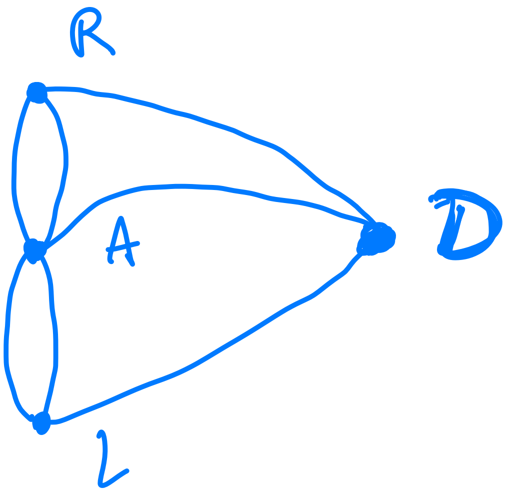


Graphs are the same!

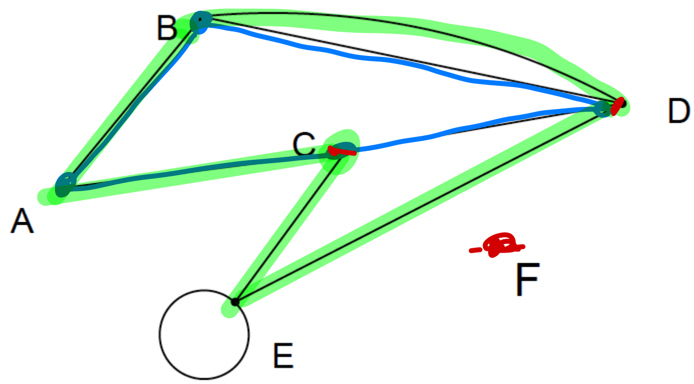
Example 3. Back in the 18th century in the Prussian city of Königsberg, a river ran through the city and seven bridges crossed the forks of the river. The river and the bridges are highlighted in the picture to the right. As a weekend amusement, townsfolk would see if they could find a route that would take them across every bridge once and return them to where they started.



Draw the graph modeling this problem.



Example 4. For next terminology consider the following graph:



→ not connected because of F

Adjacent vertices - vertices connected by an edge.

ex: A and B are adjacent vertices

B and E are not adjacent vertices

Adjacent edges - edges that share a vertex.

ex: AB and BD are adjacent edges bc they share vertex B.

AC and ED are not adj. edges.

Degree of a vertex – the number of edges at that vertex. A loop contributes twice to a degree!

deg (A) = 2 deg (C) = 3 deg(E) = 4

deg (B) = 3 deg (D) = 4 deg (F) = 0

Path – a sequence of vertices that describe a trip along the edges of a graph so that you never travel an edge more than once. The **length** of a path is the number of edges in the path.

ex: ABD is a path w/ length 2.

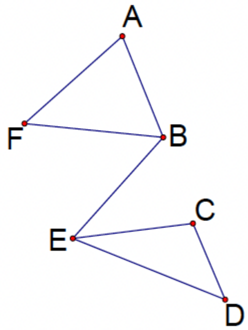
CEDB is a path w/ length 3.

Circuit – a path that starts and ends at the same vertex.

ex: BACDB

connected graph : contains a path between any two vertices.

Example 5. Consider the following graph.



Is this graph connected?

Yes!

A bridge is an edge that if it were removed from the connected graph it would make the graph disconnected.

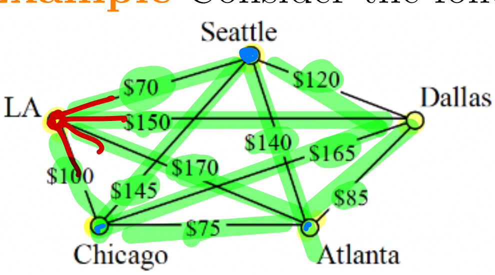
ex: EB is an example of a bridge.

AF not a bridge.

Weighted Graphs

Depending upon the problem being solved, sometimes weights are assigned to the edges. The weights could represent the distance between two locations, the travel time, or the travel cost. It is important to note that the distance between vertices in a graph does not necessarily correspond to the weight of an edge.

Example Consider the following weighted graph.



The graph shows 5 cities. The weights on the edges represent the airfare for a one-way flight between the cities.

1. How many vertices and edges does the graph have? 5 10
2. Is the graph connected? yes
3. What is the degree of the vertex representing LA? 4
4. If you fly from Seattle to Dallas to Atlanta, is that a path or a circuit?
5. If you fly from LA to Chicago to Dallas to LA, is that a path or a circuit?