## Math 140: General Multiplication Rule

Recall: independent events

$P(\underbrace{A \text{ and } B}_{\text{in a sequence}}) = P(A)\cdot P(B)$

Recall: conditional probability

$P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$

$P(* \mid \star) = \dfrac{P(* \text{ and } \star)}{P(\star)}$

## Probabilities involving "AND" continued

Independent events revisited:

1. Consider drawing two cards (one at a time) from a deck of cards **with replacement**. Let $K_{1st}$ be the event of "drawing a King the first time" and $K_{2nd}$ be the event of "drawing a King the second time". Find:

$P(K_{2nd}) = \dfrac{4}{52}$

$P(K_{2nd}|K_{1st}) = \dfrac{P(K_{2nd} \text{ and } K_{1st})}{P(K_{1st})} = \dfrac{P(K_{2nd})\cdot P(K_{1st})}{4/52}$

$= \dfrac{\frac{4}{\cancel{52}}\cdot\frac{4}{52}}{\frac{4}{\cancel{52}}} = \dfrac{4}{52}$

Event A and B are independent if and only if:

- $P(A \text{ and } B) = P(A)\cdot P(B)$
- $P(A|B) = P(A)$
- $P(B|A) = P(B)$

2. Consider drawing two cards (one at a time) from a deck **without replacement**. Let $K_{1st}$ be the event of "drawing a King the first time" and $K_{2nd}$ be the event of "drawing a King the second time". Find:

$$P(K_{2nd}|K_{1st}) = \frac{3}{51}$$

can't be broken down

$$P(K_{2nd} \mid K_{1st}) = \frac{P(K_{2nd} \text{ and } K_{1st})}{P(K_{1st})}$$

Solve for it

$$P(K_{2nd} \text{ and } K_{1st}) = P(K_{1st}) \cdot P(K_{2nd} \mid K_{1st})$$

$$= \frac{4}{52} \cdot \frac{3}{51}$$

**General Multiplication Rule**

The probability that two events $A$ and $B$ will occur in sequence is

$$P(A \text{ and } B) = P(A) \cdot P(B|A).$$

If events $A$ and $B$ are independent, then the rule can be simplified to

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

$$P(* \text{ and } \star) = P(*)\ P(\star \mid *)$$

$$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$$

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

T - event driving truck $P(T)=\frac{1}{6}$

F - event Ford. $P(F)=3/10$

**Example 1.** In a survey, 510 adults were asked if they drive a pickup truck and if they drive a Ford. The results showed that one in six adults surveyed drives a pickup truck, and three in ten adults surveyed drive a Ford. Of the adults surveyed that drive Fords, two in nine drive a pickup truck.

$$P(T \mid F) = \frac{2}{9}$$

1. Find the probability that a randomly selected adult drives a pickup truck, **given** that the adult drives a Ford.

$$P(T \mid F) = \frac{2}{9}$$

2. Are the events "driving a Ford" and "driving a pickup truck" independent or dependent? Explain.

Check if $P(T \mid F) \stackrel{?}{=} P(T)$

$$\frac{2}{9} \neq \frac{1}{6}$$

not independent!

3. Find the probability that a randomly selected adult drives a Ford pick up truck.

$$P(F \text{ and } T) = P(F)\,P(T \mid F)$$

$$= \frac{\cancel{3}}{10} \cdot \frac{2}{\cancel{9}\,3} = \frac{2}{30} = \frac{1}{15}$$

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I: $P(F \text{ and } T) \neq P(F) \cdot P(T)$ T and F not indepent

II $P(F \text{ and } T) = P(T) \cdot \underbrace{P(F \mid T)}_{\text{don't know}}$