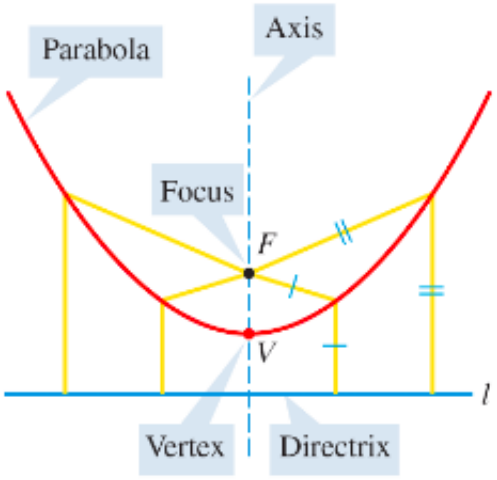
1. Parabolas

In this section we study parabolas from a geometric rather than an algebraic point of view.
We begin with the geometric definition of a parabola and show how this leads to the algebraic formula that we are already familiar with.

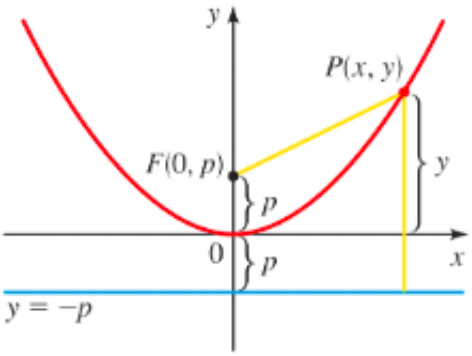
GEOMETRIC DEFINITION OF A PARABOLA

A **parabola** is the set of all points in the plane that are equidistant from a fixed point F (called the **focus**) and a fixed line l (called the **directrix**).



2. Deriving the equation of the parabola

We will concentrate on parabolas that are situated with the vertex at the origin and that have a vertical or horizontal axis of symmetry.



PARABOLA WITH VERTICAL AXIS

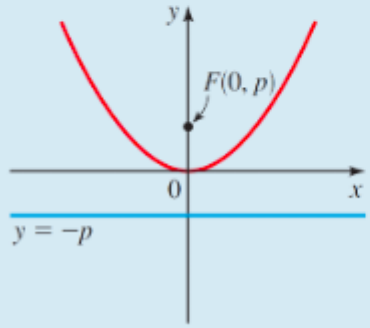
The graph of the equation

$$x^2 = 4py$$

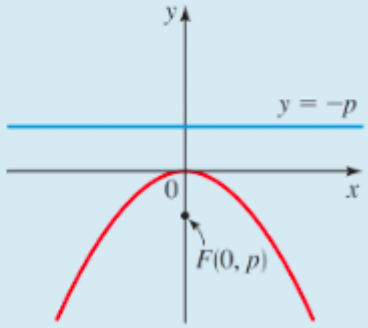
is a parabola with the following properties.

VERTEX	$V(0, 0)$
FOCUS	$F(0, p)$
DIRECTRIX	$y = -p$

The parabola opens upward if $p > 0$ or downward if $p < 0$.



$x^2 = 4py$ with $p > 0$



$x^2 = 4py$ with $p < 0$

PARABOLA WITH HORIZONTAL AXIS

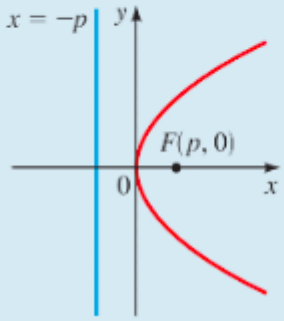
The graph of the equation

$$y^2 = 4px$$

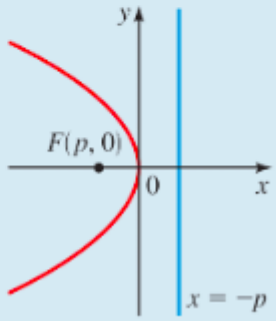
is a parabola with the following properties.

VERTEX	$V(0, 0)$
FOCUS	$F(p, 0)$
DIRECTRIX	$x = -p$

The parabola opens to the right if $p > 0$ or to the left if $p < 0$.

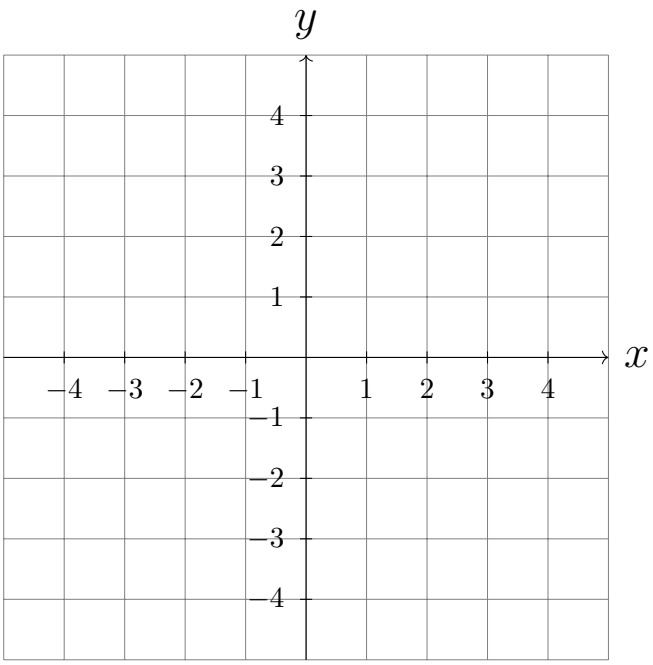


$y^2 = 4px$ with $p > 0$

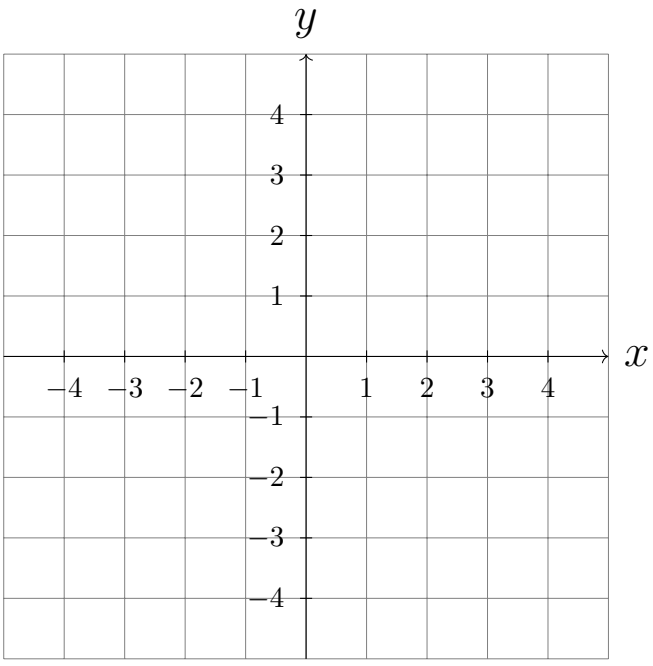


$y^2 = 4px$ with $p < 0$

Example 1: Find an equation for the parabola with vertex $V(0,0)$ and focus $F(0,2)$, and sketch its graph.



Example 2: A parabola has the equation $6x + y^2 = 0$.
Find the focus, focal diameter and directrix of the parabola and draw the graph.



3. Graphing hyperbolas with different center

The following summarizes the equation and features of hyperbolas centered at (h, k).

Example 3: Identify the vertex, focus, directrix, axis of symmetry, lactus rectum of the parabola $y^2 - 6y - 4x + 9 = 0$. Draw the parabola.