p and q → conjunction $p \wedge q$; p or q disjunction $p \vee q$

$\sim p$: not p

Conditionals and Equivalence guided notes

Conditional: The statement "If p, then q." Symbolized by $p \to q$ is called a conditional statement. This is also called an implication.

p is called the antecedent

q is called the consequent

Example: If $\underbrace{\text{I get my work done by 4 pm}}_{p}$, then $\overbrace{\text{we will go for a walk}}^{q}$.

"I get my work done by 4 pm." is the antecedent.
"We will go for a walk." is the consequent.

There are four possibilities:

1. I get my work done by 4 pm; We go for a walk.
2. I get my work done by 4 pm; We don't go for a walk.
3. I don't get my work done by 4 pm; We go for a walk.
4. I don't get my work done by 4 pm; We don't go for a walk.

When is the promise broken? for option 2.

If T then F.

Truth Table for the Conditional: → If p then q or p implies q

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

they are vacuosly true.

Important note:

The conditionoe is false only when the promise is broken. (T → F)

Hidden Conditional Statements:

Examples: Re-write using the if…then…connective.

1. We will go for a walk if I get my work done by 4 pm.

   If I get my work done by 4 pm, then we will go for a walk.

2. Every picture tells a story.

   If it's a picture, then it tells a story.

3. No guinea pigs are scholars.

   If it's a guinea pig, then it's not a scholar.

4. It is difficult to study when you are distracted.

   If you are distracted, then it's difficult to study.

Example: Translate the following statements into symbols given that

p : I drive to work.

q : The dog eats my shoes.

r : Milk costs $3.50 per gallon.

1) $\sim p \to r$

2) $p \vee (\sim q \to r)$

1. If I do not drive to work, then milk costs $3.50 per gallon.
2. I drive to work or if the dog does not eat my shoes then milk costs $3.50 per gallon.

- Translate into words: $(\sim r \vee \sim p) \to \sim q$ using the simple statements above.

If milk doesn't cost 3.50 or I don't drive to work, then the dog does not eat my shoes.

Examples: Determine the truth value of the conditional.

1. $F \to (4 \neq 7)$

   $F \to T$ $\boxed{\text{True}}$

2. $(4 = 11 - 7) \to (8 < 0)$

   $T \to F$ $\boxed{\text{False}}$

3. Let the statement p be false, q be true, and r be false. What is the truth value of $(\sim r \vee p) \to q$?

   $(\sim r \vee p) \to q$ $\quad (T \vee F) \to T$

   $T \to T$ TRUE

Example: Explain why in the statement $p \to [(\sim r \vee q) \wedge (\sim q \vee p)]$ the statement is true if all we know is that p is false?

consequent

$F \to \ldots$

Construct the truth table for the statement: $(p \wedge q) \to (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$T \to F$

implication false only in the case $T \to F$

Tautology: A statement that is always true no matter the truth values of its components is called a tautology.

Example: Construct the truth table for the statement: $\left[(p \wedge r) \wedge (p \wedge q)\right] \rightarrow p$

p	q	r	$p \wedge r$	$p \wedge q$	$(p \wedge r) \wedge (p \wedge q)$	
T	T	T	T	T	T	T
T	T	F	F	T	F	T
T	F	T	T	F	F	T
F	T	T	F	F	F	T
T	F	F	F	F	F	T
F	T	F	F	F	F	T
F	F	T	F	F	F	T
F	F	F	F	F	F	T

Equivalent Statements: Two statements are equivalent if they have the same truth value for every possible situation.

Notation for equivalent statements: next class

Example: Show that $p \rightarrow q \equiv \sim p \vee q$

Example: Show that

Fill in the following truth table:

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$

Definitions: For the conditional $p \to q$

Converse: $q \to p$

Inverse: $\sim p \to \sim q$

Contrapositive: $\sim q \to \sim p$

Equivalences:

Example: Write the following for the conditional statement: If you build it, he will come.

Converse:

Inverse:

Contrapositive:

Example: If n^2 is odd, then n is odd. Write the contrapositive of this conditional.

Alternative ways to say “If p, then q.

Example: If I live in Chicago, then I live in Illinois.

1. Living in Chicago is sufficient for living in Illinois.
2. Living in Illinois is necessary for living in Chicago.
3. I live in Chicago only if I live in Illinois.
4. All people who live in Chicago live in Illinois.
5. I live in Chicago implies I live in Illinois.
6. I live in Illinois if I live in Chicago.
7. I live in Illinois whenever I live in Chicago.

Biconditional: p if and only if q (p is necessary and sufficient for q)

Symbol:

Meaning:

What is the truth table for the biconditional?

True or False:

1. $3+1\neq 6$ if and only if $8\neq 8$.

2. $6\times 2=14$ if and only if $9+7\neq 16$.

3. $12\times 1\neq 13$ if and only if $9\neq 7$.

Construct the truth table for the statement:

$$\left(p\wedge\sim q\right)\leftrightarrow\left[\left(\sim p\vee q\right)\rightarrow p\right]$$