**Math 142: Monday, Oct 21st, Day 17 Notes**

Agenda for Monday, Oct 21st 2024



## Coming up

- HW 13 posted & due next Monday
- Student Hours Tuesday 2:15-3:30pm in C162

**Recall:** Factorial notation

$3! = 3 \cdot 2 \cdot 1 = 6$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

In English we use the word "combination" loosely. We say things like:

- "My fruit salad is a combination of apples, grapes and bananas"
  AGB is the same GAB (order does not matter)
- "The combination to the safe is 472".
  $472 \neq 427$ order matters

When the order doesn't matter, it is a **Combination**.
When the order does matter it is a **Permutation**.

**Permutations and Combinations:** Consider having the letters A, B, C, D. Let's choose two of the letters.

In how many different ways can we arrange two of the 4 letters? In how many different ways can we combine two of the 4 letters?

AB BA CA DA
AC BC CB DB
AD BD CD DC

\# of combinations (order does not matter) = 6

\# of permutations (order matters) = 12

$n = 4$ letters A, B, C, D

**Permutations:** We use permutations when $n$ is the number of things to choose from, and we choose $r$ of them, no repetitions, order matters.

$r = 2$

**Formula and Notation**:

$$_nP_r = \frac{n!}{(n-r)!}$$

ex: $n = 4$ letters
$r = 2$ letters

$$_4P_2 = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}} = 12$$

**Calculator instructions**:

put $n$ then press Math → PROB → $nPr$ → enter $r$

**Example 1.** A volleyball team has 10 players, how many 6 player starting line up are possible?

$$_{10}P_6 = 151,200$$

**Combinations** We use combination when $n$ is the number of things to choose from, and we choose $r$ of them, no repetition, order doesn't matter.

**Formula and Notation**:

$$_nC_r = \frac{n!}{(n-r)!\, r!}$$

$n = 4$ letters
$r = 2$

$$_4C_2 = \frac{4!}{(4-2)!\, 2!} = \frac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2 \cdot 1} \cdot \cancel{2} \cdot 1} = 6$$

**Calculator instructions**:

$n$ → Math → PROB → $nCr$ → enter $r$

**Example 2.** Four lawyers are to be selected from a group of 30 lawyers to work on a project.

$r = 4$ $\quad$ $n = 30$

1. How many different ways can the lawyers be selected to work on the project?

order does not matter

$$_{30}C_4 = 27405$$

2. In how many ways can the group of 4 be selected if a certain lawyer must work on the project?

Maxwell : $_{30}P_4$ = 657720

Mario : $_{30}C_3$ =

Correct : $_{29}C_3$ = 3654

**Example 3.** In a student club with 9 sophomores and 11 freshman members, a 5- member committee will be randomly chosen. How many ways can a committee be chosen with 3 sophomores and 2 freshmen?

9 soph

11 Fresh

$\underbrace{_9C_3}_{\text{3 soph out 9}} \cdot {_{11}C_2} = 84 \cdot 55 = 4620$

## Discrete Random variables

**Example 4.** Consider tossing a coin 3 times. The sample space for the toss of 3 coins is

S={TTT, THH, HTH, HHT, HTT, THT, TTH, HHH }

Lets say we are interested in the **number of heads** you get when you toss the coin 3 times.

X = the number of heads that appear when you perform the experiment

It is called a random variable

What are the possible values for $X$?

X = {0, 1, 2, 3}

Make a table with all the possible values of X and their corresponding probabilities.

| X | P(X) |
|---|---|
| 0 | 1/8 |
| 1 | 3/8 |
| 2 | 3/8 |
| 3 | 1/8 |

Recall: weighted mean

90 45%

80 15%

⋮

= 90(.45) + 80(.15) + ...

We call X a **random variable** and the table is called a **probability distribution function(pdf)**.

**Definitions:**

Random Variable – a variable whose numeric value is determined by outcome of a random experiment. We have two types of random variables:

→ 0 TTT

1. Continuous random variables X: results of measurements (take on any real number: 1.01, 1.00911)
2. Discrete random variables X: usually results of "counts"
   - take on a countable number of distinct values
   -2, -1, 0, 1, 2, 3 (count them: 6 values)

Probability distribution function (pdf)– lists the probabilities for each outcome of the random variable:

We use capital letter for random variables: X, Y, Z.

**Example 5.** Determine if the following random variables is continuous or discrete.

X: length of time it takes to get to work → continuous

Y: number of tornadoes in the month of June → discrete

X | P(X)

Rules of a probability distribution:

1. each probability is between 0 and 1
2. all probabilities add to 1.

**Example 6.** Find the missing probabilities that will make this distribution a proper probability distribution.

X: number of policies sold by insurance salespeople in a day.

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P(x) | 0.05 | ? .15 | 0.23 | 0.21 | 0.17 | 0.11 | 0.08 |

add up to .85

- Find P(x=1) = 1 − .85 = .15

- Find P($x > 4$) and interpret it.

$P(x>4) = P(x=5) + P(x=6) = .11 + .08 = .19$

The chance of a salesperson or selling more than 4 policies in a day is 19%..

**Example 7.** Create a probability distribution where X is the resulting sum in rolling a pair of standard dice.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1, 1) | (1, 2) | (1, 3) | (1, 4) | (1, 5) | (1, 6) |
| 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) | (2, 6) |
| 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) | (3, 6) |
| 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) | (4, 6) |
| 5 | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) | (5, 6) |
| 6 | (6, 1) | (6, 2) | (6, 3) | (6, 4) | (6, 5) | (6, 6) |

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| X | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P(X) | 1/36 | 2/36 | 3/36 | 4/36 | 5/36 | 6/36 | 5/36 | 4/36 | 3/36 | 2/36 | 1/36 |

$P(\text{sum} \geq 10) = P(X \geq 10) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{6}{36} = \frac{1}{6}$

## Mean, Expected Value and Standard Deviation

| $X$ | $P(X)$ |
|---|---|
| $x_1$ | $P(x_1)$ |
| $x_2$ | $P(x_2)$ |
| $\vdots$ | $\vdots$ |
| $x_n$ | $P(x_n)$ |

Let X be a random variable and $x_1, x_2, \cdots x_n$ be the list of possible outcomes for X.

- Expected Value $E(X)$ or mean $\mu$ of a discrete probability distribution:

  $$\mu = E(X) = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_2 P(x_n)$$

- Standard deviation of probability distribution,$\sigma$

  $\sigma$

- Variance: $\sigma^2$

By calculator place x into $L_1$ and P(x) into $L2$ , 1-var Stat $L_1$, $L_2$

**Example 8.** A men's soccer team plays soccer zero, one, or two days a week. The probability that they play zero days is 0.2, the probability that they play one day is 0.5, and the probability that they play two days is 0.3. Find the long-term average or expected value, $\mu$, of the number of days per week the men's soccer team plays soccer.

| $X$ | $P(X)$ |
|---|---|
| 0 | 0.2 |
| 1 | 0.5 |
| 2 | 0.3 |

$$\mu = E(X) = 0\cdot(0.2) + (1)(0.5) + (2)(.3)$$
$$= 0 + 0.5 + .6$$
$$= 1.1 \text{ days}$$

$\mu = E(X) = 1.1 = (\bar{x} \text{ in calc})$

$\sigma = 0.7 = (\sigma_x \text{ in calc})$

variance $= (0.7)^2 = 0.49$

**Example 9.** A charity organization is selling \$5 raffle tickets as part of a fund-raising program. The first prize is a trip to Mexico valued at \$3450, and the second prize is a weekend spa package valued at \$750. The remaining 10 prizes are \$25 gift cards. The number of tickets sold is 6000. Find the expected value of a single raffle ticket.

next class

**Practice:**

1. Determine if the following is a probability distribution. If not, determine which property is not met.

   A quality inspector checked for imperfections in rolls of fabric for 1 week. The random variable X represents the number of imperfections found.

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| P(x) | $\frac{3}{4}$ | $\frac{1}{10}$ | $\frac{1}{20}$ | $\frac{1}{25}$ | $\frac{1}{50}$ | $\frac{1}{100}$ |

2. The random variable X represents the number of credit cards that adults have along with the corresponding probabilities. Find the mean (expected value) and standard deviation.

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P(x) | 0.06 | .58 | 0.21 | 0.08 | 0.05 | 0.01 | 0.01 |

3. A 40-year-old man in the U.S. has a 0.242% risk of dying during the next year . An insurance company charges \$275 for a life-insurance policy that pays a \$100,000 death benefit. What is the expected value for the person buying the insurance?