Day 11 Notes

3.1 and 3.2 Exponential and Logarithmic functions - Review

1. Section 3.1 Exponential Functions

Exponential and logarithmic functions represent two fundamental mathematical relationships that are inverses of each other. Exponential functions, written in the form $f(x) = b^x$ where $b > 0$ and $b \neq 1$, model growth and decay phenomena where a quantity changes at a rate proportional to its current value. These functions appear in numerous real-world applications, from compound interest and population growth to radioactive decay.

An exponential function is a function of the form $f(x) =$ ____, where the base b ____ and b ____
Note: The independent variable x is in the ______________ !

Ex:

The exponential function with base e is $f(x) =$ ____, where $e = \left(1 + \frac{1}{n}\right)^n$ as $n \to \infty$. Note $e \approx$ ____.

Calculator buttons:

Recall rules of exponents:

$a^m \cdot a^n = a^{m+n}$	$(a^m)^n = a^{mn}$	$a^0 = 1$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
$\frac{a^m}{a^n} = a^{m-n}$	$(ab)^n = a^n b^n$	$a^{-n} = \frac{1}{a^n}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 1. Use a calculator(if needed) to evaluate the function at the indicated value of x. Round to three decimal places.

a. $f(x) = 5^x$ at $x = 2$

b. $f(x) = 2^x$ at $x = -3$

c. $f(x) = \left(\frac{1}{4}\right)^x$ at $x = 0$

d. $f(x) = 2.3^x$ at $x = -1$

Example 2. Graph the function and write the domain and range in interval notation.

<table>
<tr><td>1) $f(x) = 3^x$</td><td>2) $f(x) = \left(\frac{1}{3}\right)^x$</td></tr>
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<table>
<tr><td>x</td><td>$y = 3^x$</td></tr>
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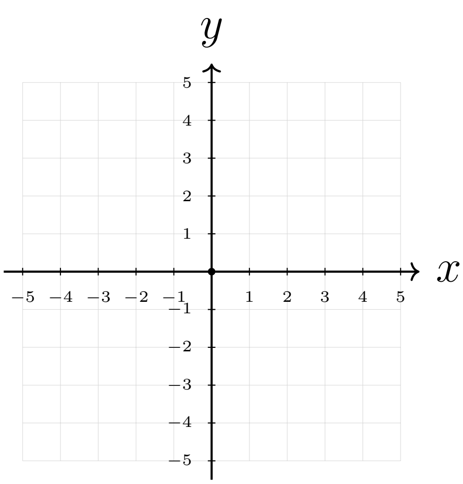

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<tr><td>x</td><td>$y = \left(\frac{1}{3}\right)^x$</td></tr>
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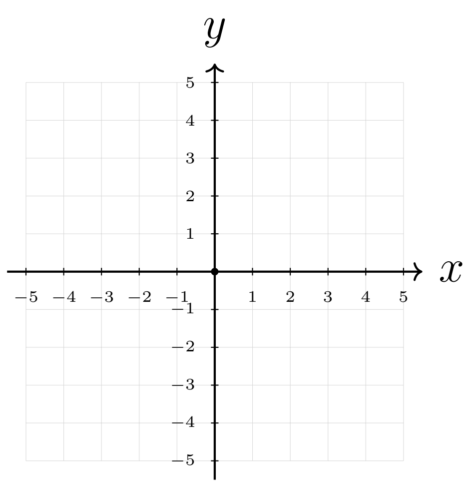

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<tr><td>H.A.</td><td>H.A.</td></tr>
<tr><td>domain</td><td>domain</td></tr>
<tr><td>range</td><td>range</td></tr>
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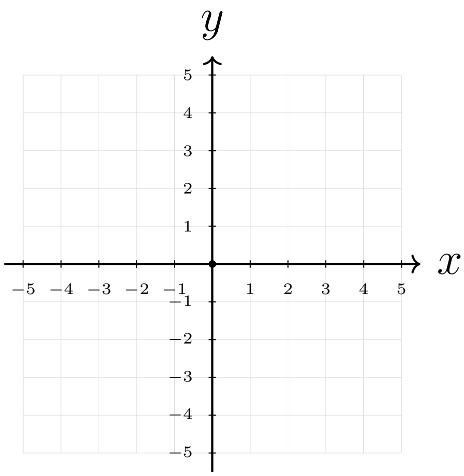
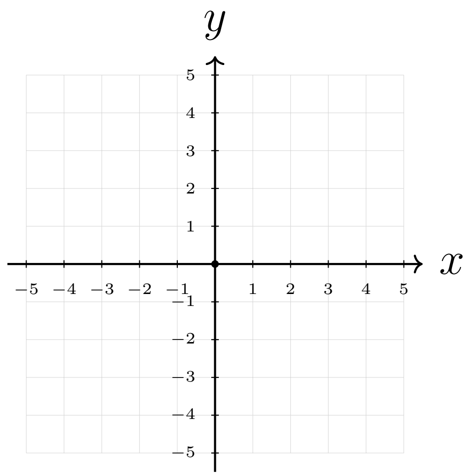
Example 3. Use the graph of the parent function to describe the transformation that yields the graph of g.

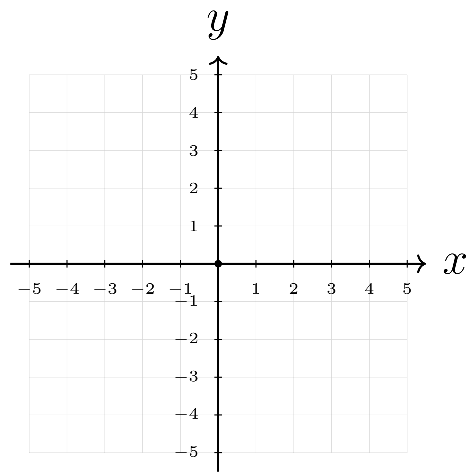
$g(x) = 2^{x+1} - 3$



y
x

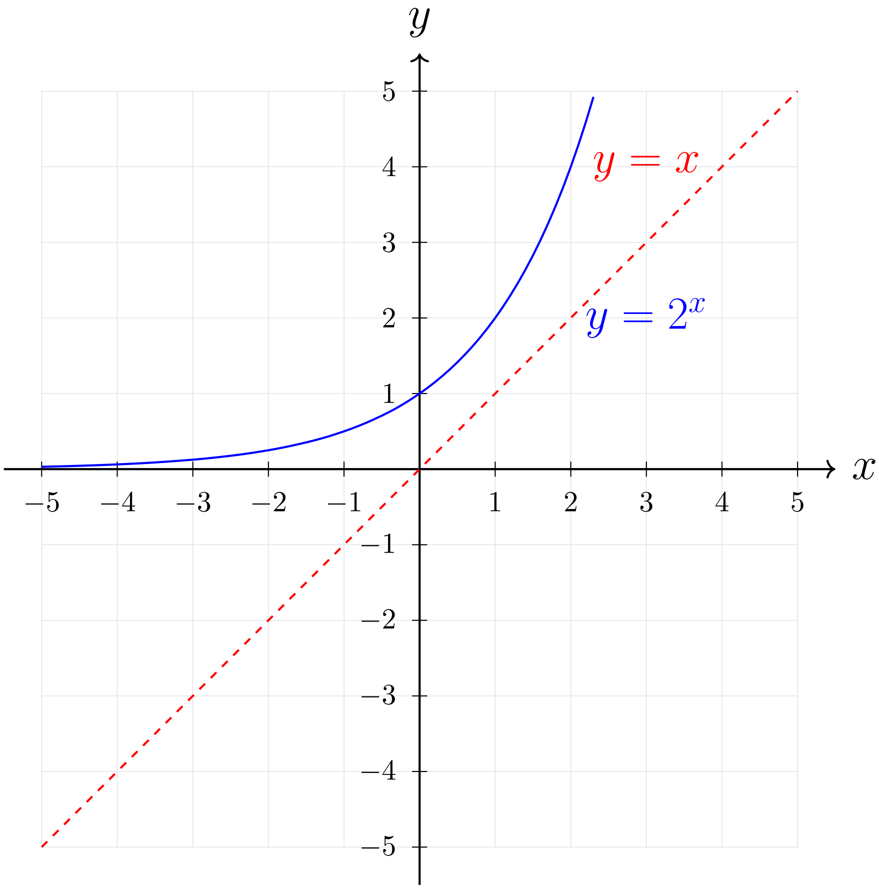


y
x



y
x

The inverse of the exponential function $f(x) = b^x$ is called **logarithmic function with base a**



Notation for logarithmic function:

Domain:

Range:

Asymptote:

A logarithmic function is a function of the form $f(x) =$ ______, where x ____, and the base b ____ and b ____.

Special Logarithms:
1) common log:
2) natural log:

Example 4. Write the logarithmic equation in exponential form.

1. $\log_2 8 = 3$
2. $\log_{10} 7 = x$
3. $\log_{1/2} y = 5$
4. $\ln 1 = 0$

Example 5. Write the exponential form in logarithmic equation.

1. $5^x = 23$
2. $8^{-2} = \dfrac{1}{64}$
3. $b^n = 9$

Example 6. Use the definition of logarithmic function to evaluate the function at the indicated value of x without using a calculator.

1. $f(x) = \log_5 x$ at $x = 25$
2. $f(x) = \log_2 x$ at $x = 8$
3. $f(x) = \log x$ at $x = 100000$
4. $f(x) = \ln x$ at $x = e^9$
5. $f(x) = \log x$ at $x = 0.001$
6. $f(x) = \ln x$ at $x = \frac{1}{e^5}$

7. $f(x) = \log_4 x$ at $x = 1$

8. $f(x) = \log_2 x$ at $x = 0$

9. $f(x) = \log_2 x$ at $x = -16$

Example 7. Use a calculator to evaluate the function at the indicated value of x.

1. $f(x) = \log x$ at $x = 123$

2. $f(x) = \ln x$ at $x = 2$

Basic Properties of Logarithms with base b, where $b > 0$, $b \neq 1$, and $x > 0$

1) $\log_b 1 =$ because

2) $\log_b b =$ because

3) $\log_b b^x =$ because

4) $b^{\log_b x} =$ because

EX Use the properties of logarithms to simplify the expression, without using a calculator.

1) $\log_3 3^{5x} =$

2) $\log_{\sqrt{2}} 1 =$

3) $6\log_4(1/4) =$

4) $4^{\log_4(5x+y)} =$

EX Use the properties of natural logarithms to rewrite the expression, without using a calculator.

1) $\ln e^2 =$ 3) $\ln e =$

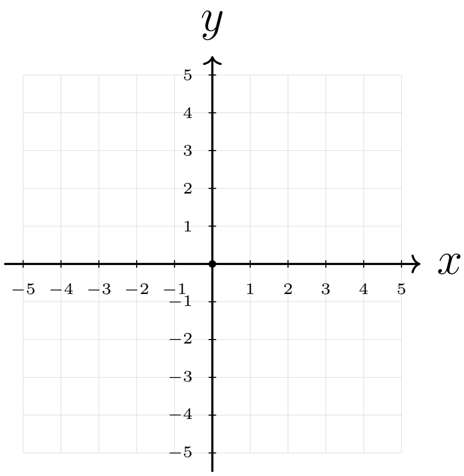
2) $5 \ln 1 =$ 4) $e^{\ln(4.2)} =$

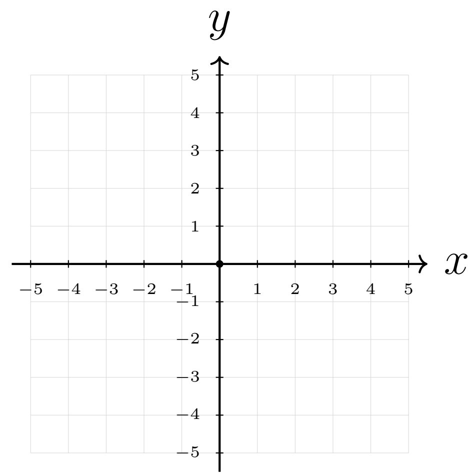
Example 8. Find the domain in interval notation.

1. $y = \log_{1/2}(7 - 3x)$

2. $y = \log_2(x^2 - x - 12)$

Graphs of logarithmic functions





Example 9. Use the graph of the parent function to describe the transformation that yields the graph of g. Then find the domain and range in interval notation, the vertical asymptote and the x-intercept.
$g(x) = -\log_2(x-1)$