Conditionals and Equivalence guided notes

Conditional: The statement "If p, then q." Symbolized by $p \rightarrow q$ is called a conditional statement. This is also called an ___________________________.

p is called the ____________________________

q is called the _____________________________

Example: If I get my work done by 4 pm, then we will go for a walk.

"I get my work done by 4 pm." is the __________________________.
"We will go for a walk." is the _______________________________.

There are four possibilities:

1. I get my work done by 4 pm; We go for a walk.
2. I get my work done by 4 pm; We don't go for a walk.
3. I don't get my work done by 4 pm; We go for a walk.
4. I don't get my work done by 4 pm; We don't go for a walk.

When is the promise broken?

Truth Table for the Conditional:

Important note:

Hidden Conditional Statements:

Examples: Re-write using the if…then…connective.

1. We will go for a walk if I get my work done by 4 pm.
2. Every picture tells a story.
3. No guinea pigs are scholars.
4. It is difficult to study when you are distracted.

Example: Translate the following statements into symbols given that
p: I drive to work.
q: The dog eats my shoes.
r: Milk costs \$3.50 per gallon.

1. If I do not drive to work, then milk costs \$3.50 per gallon.
2. I drive to work or if the dog does not eat my shoes then milk costs \$3.50 per gallon.

Translate into words: $(\sim r \vee \sim p) \to \sim q$ using the simple statements above.

Examples: Determine the truth value of the conditional.

1. $F \to (4 \neq 7)$
2. $(4 = 11 - 7) \to (8 < 0)$
3. Let the statement p be false, q be true, and r be false. What is the truth value of $(\sim r \vee p) \to q$?

Example: Explain why in the statement $p \to [(\sim r \vee q) \wedge (\sim q \vee p)]$ the statement is true if all we know is that p is false?

Construct the truth table for the statement: $(p \wedge q) \to (p \vee q)$

Tautology: A statement that is always true no matter the truth values of its components is called a tautology.

Example: Construct the truth table for the statement: $\left[(p \wedge r) \wedge (p \wedge q)\right] \rightarrow p$

Equivalent Statements: Two statements are equivalent if they have the same truth value for every possible situation.

Notation for equivalent statements:

Example: Show that $p \rightarrow q \equiv \sim p \vee q$

Example: Show that $\sim (p \vee q) \equiv \sim p \wedge \sim q$

DeMorgan's Laws:

1. $\sim(p \vee q) \equiv \sim p \wedge \sim q$

2. $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Examples: Write the negation of the following statements using DeMorgan's Laws:

1. The cat is brown and the cat is fluffy.

2. The lawyer or the client appeared in court.

3. $3 < 10$ or $7 \neq 2$

Example: Re-write this conditional statement as an equivalent disjunction.
If the check is in the mail, then I will be surprised.

The negation of the conditional $\sim(p \rightarrow q) \equiv p \wedge \sim q$

Example: Write the negation of this conditional statement as a conjunction.
If the check is in the mail, then I will be surprised.

Fill in the following truth table:

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$

Definitions: For the conditional $p \to q$

Converse: $q \to p$

Inverse: $\sim p \to \sim q$

Contrapositive: $\sim q \to \sim p$

Equivalences:

Example: Write the following for the conditional statement: If you build it, he will come.

Converse:

Inverse:

Contrapositive:

Example: If n^2 is odd, then n is odd. Write the contrapositive of this conditional.

Given that the conditional is $p \to q$; converse: $q \to p$; inverse: $\sim p \to \sim q$; contrapositive: $\sim q \to \sim p$

Complete the table below:

	Converse	Inverse	Contrapositive
Converse			
Inverse			
Contrapositive			

Alternative ways to say "If p, then q.

Example: If I live in Chicago, then I live in Illinois.

1. Living in Chicago is sufficient for living in Illinois.
2. Living in Illinois is necessary for living in Chicago.
3. I live in Chicago only if I live in Illinois.
4. All people who live in Chicago live in Illinois.
5. I live in Chicago implies I live in Illinois.
6. I live in Illinois if I live in Chicago.
7. I live in Illinois whenever I live in Chicago.

Biconditional: p if and only if q (p is necessary and sufficient for q)

Symbol:

Meaning:

What is the truth table for the biconditional?

True or False:

1. $3+1\neq 6$ if and only if $8\neq 8$.
2. $6\times 2=14$ if and only if $9+7\neq 16$.
3. $12\times 1\neq 13$ if and only if $9\neq 7$.

Construct the truth table for the statement:

$$(p\wedge \sim q)\leftrightarrow\left[(\sim p\vee q)\rightarrow p\right]$$