

1. Exponential Functions

Exponential and logarithmic functions represent two fundamental mathematical relationships that are inverses of each other. Exponential functions, written in the form $f(x) = b^x$ where $b > 0$ and $b \neq 1$, model growth and decay phenomena where a quantity changes at a rate proportional to its current value. These functions appear in numerous real-world applications, from compound interest and population growth to radioactive decay.

An exponential function is a function of the form $f(x) = ______$, where the base b $______$ and b $______$

Note: The independent variable x is in the $______$!

Ex:

The exponential function with base e is $f(x) = ______$, where $e = \left(1 + \frac{1}{n}\right)^n$ as $n \rightarrow \infty$. Note $e \approx ______$.

Calculator buttons:

Recall rules of exponents:

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Example 1. Use a calculator(if needed) to evaluate the function at the indicated value of x . Round to three decimal places.

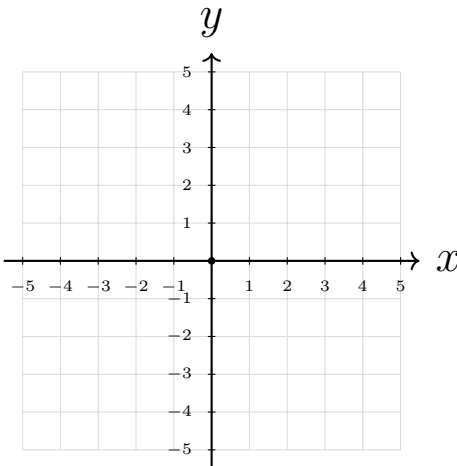
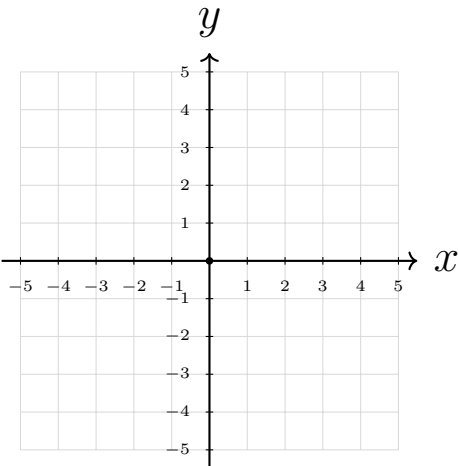
a. $f(x) = 5^x$ at $x = 2$

c. $f(x) = \left(\frac{1}{4}\right)^x$ at $x = 0$

b. $f(x) = 2^x$ at $x = -3$

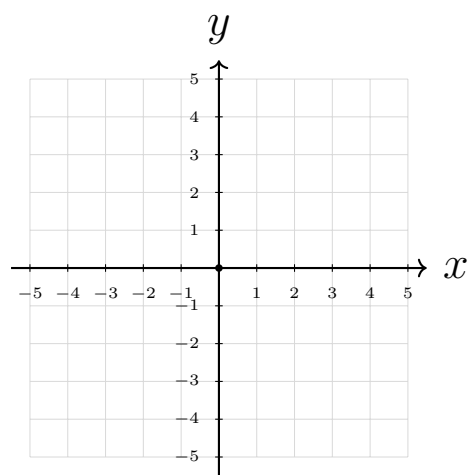
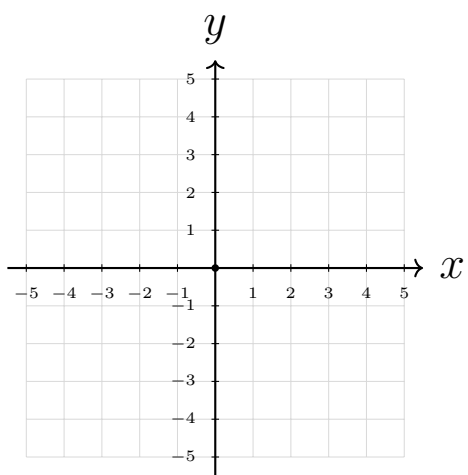
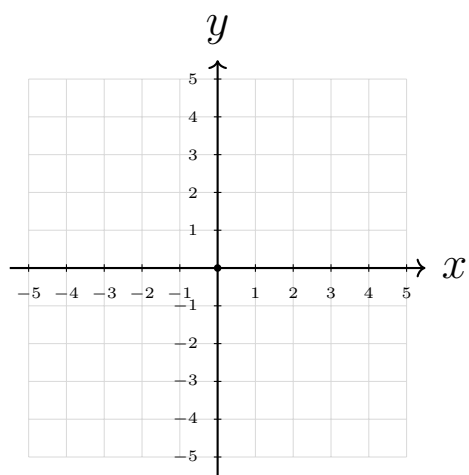
d. $f(x) = 2.3^x$ at $x = -1$

Example 2. Graph the function and write the domain and range in interval notation.

1) $f(x) = 3^x$	2) $f(x) = \left(\frac{1}{3}\right)^x$																
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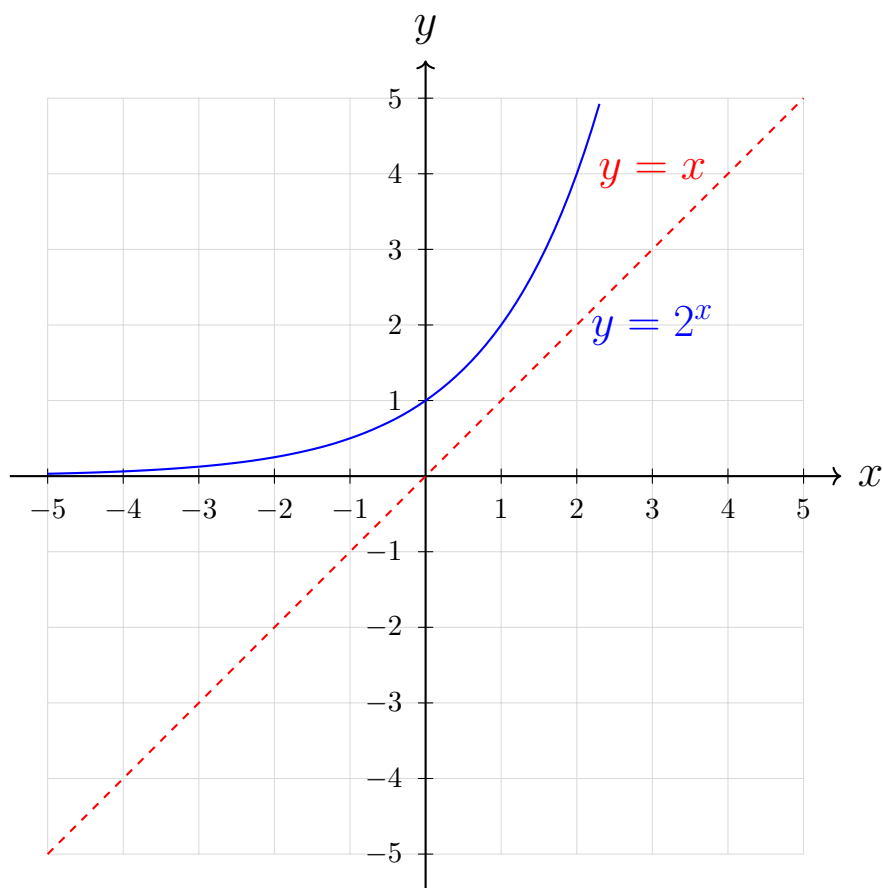
Example 3. Use the graph of the parent function to describe the transformation that yields the graph of g .

$g(x) = 2^{x+1} - 3$



2. Section 6.3 Logarithmic Functions

The inverse of the exponential function $f(x) = b^x$ is called **logarithmic function with base a**



Notation for logarithmic function:

Domain:

Range:

Asymptote:

A logarithmic function is a function of the form $f(x) = \log_b(x)$, where $x > 0$, and the base $b > 0$ and $b \neq 1$.

Special Logarithms:

1) common log:

2) natural log:

Example 4. Write the logarithmic equation in exponential form.

1. $\log_2 8 = 3$

2. $\log_{10} 7 = x$

3. $\log_{1/2} y = 5$

4. $\ln 1 = 0$

Example 5. Write the exponential form in logarithmic equation.

1. $5^x = 23$

2. $8^{-2} = \frac{1}{64}$

3. $b^n = 9$

Example 6. Use the definition of logarithmic function to evaluate the function at the indicated value of x without using a calculator.

1. $f(x) = \log_5 x$ at $x = 25$

2. $f(x) = \log_2 x$ at $x = 8$

3. $f(x) = \log x$ at $x = 100000$

4. $f(x) = \ln x$ at $x = e^9$

5. $f(x) = \log x$ at $x = 0.001$

6. $f(x) = \ln x$ at $x = \frac{1}{e^5}$

7. $f(x) = \log_4 x$ at $x = 1$

8. $f(x) = \log_2 x$ at $x = 0$

9. $f(x) = \log_2 x$ at $x = -16$

Example 7. Use a calculator to evaluate the function at the indicated value of x .

1. $f(x) = \log x$ at $x = 123$

2. $f(x) = \ln x$ at $x = 2$

Basic Properties of Logarithms with base b , where $b > 0$, $b \neq 1$, and $x > 0$

1) $\log_b 1 =$ because

2) $\log_b b =$ because

3) $\log_b b^x =$ because

4) $b^{\log_b x} =$ because

EX Use the properties of logarithms to simplify the expression, without using a calculator.

1) $\log_3 3^{5x} =$ 3) $6 \log_4(1/4) =$

2) $\log_{\sqrt{2}} 1 =$ 4) $4^{\log_4(5x+y)} =$

EX Use the properties of natural logarithms to rewrite the expression, without using a calculator.

1) $\ln e^2 =$ 3) $\ln e =$

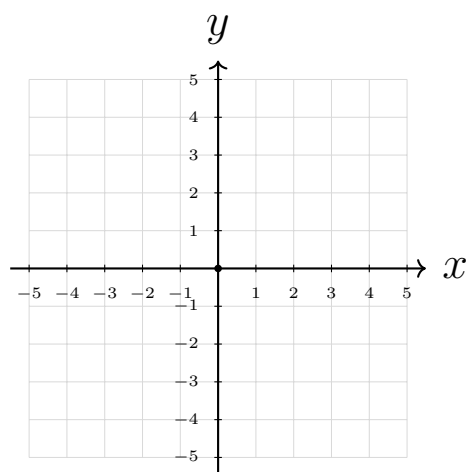
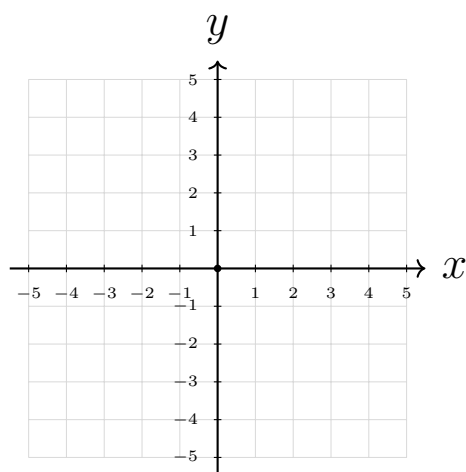
2) $5 \ln 1 =$ 4) $e^{\ln(4.2)} =$

Example 8. Find the domain in interval notation.

1. $y = \log_{1/2}(7 - 3x)$

2. $y = \log_2(x^2 - x - 12)$

Graphs of logarithmic functions



Example 9. Use the graph of the parent function to describe the transformation that yields the graph of g . Then find the domain and range in interval notation, the vertical asymptote and the x-intercept.

$$g(x) = -\log_2(x - 1)$$

