**Day 3 Notes**
**9.4 Series and their Notation**

## 1. Series, partial sums and Sigma notation

If we add the terms of a sequence we obtain a **series**.

### Sequence:

ex:

### Series:

ex:

### Partial sum:

ex:

ex:

ex:

### Sigma notation: [1]

[1] We use **Sigma notation** for both infinite series and finite series.

**Example 1.** Decide if we are working with a finite sum or a series. Then write the sum using summation notation.

1. $5 + 10 + 15 + 20 + 25 + 30$

2. $\dfrac{\sqrt{3}}{1} + \dfrac{\sqrt{4}}{2} + \dfrac{\sqrt{5}}{3} + \cdots + \dfrac{\sqrt{n+2}}{n}$

3. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

**Example 2.** Write the terms for the sum and evaluate the sum.
$\sum_{n=1}^{5} 2n + 3$

**PARTIAL SUMS OF AN ARITHMETIC SEQUENCE**

For the arithmetic sequence given by $a_n = a + (n-1)d$, the ***n*th partial sum**

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \cdots + [a + (n-1)d]$$

is given by either of the following formulas.

**1.** $S_n = \frac{n}{2}[2a + (n-1)d]$ **2.** $S_n = n\left(\frac{a + a_n}{2}\right)$

**Example 4:** a) Find the sum of the first 100 numbers.

b) Find the sum of the first 50 odd numbers.

c) Sum the first 25 terms of the sequence $3, -1, -5, -9, \cdots$

## 3. Partial Sums of Geometric Sequences

The partial sum $s_n$ of a geometric sequence is given by the formula below.

**PARTIAL SUMS OF A GEOMETRIC SEQUENCE**

For the geometric sequence defined by $a_n = ar^{n-1}$, the ***n*th partial sum**

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} \qquad r \neq 1$$

is given by

$$S_n = a\frac{1 - r^n}{1 - r}$$

**Example 7:** Find the following partial sum of a geometric sequence:
$1 + 4 + 16 + \cdots + 4096$

## 4. Infinite Geometric Series

An infinite geometric series is a series of the form
$a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} + \cdots$.

**SUM OF AN INFINITE GEOMETRIC SERIES**

If $|r| < 1$, then the infinite geometric series

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \cdots$$

converges and has the sum

$$S = \frac{a}{1 - r}$$

If $|r| \geq 1$, the series diverges.

**Example 8:** Determine whether the infinite geometric series is convergent or divergent. If it is convergent, find its sum.

(a) $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \cdots$

(b) $1 + \frac{7}{5} + \frac{49}{25} + \cdots$