Math 140: Chapter 5 Logic

Recall : p
~p negation

Logical Connectives

The words **"and"**, **"or"**, **"if...then"**, **"if and only if"** are examples of logical connectives. They are words that can be used to connect two simple statements to form a more complicated compound statement.
Example of compound statement:

- "If I pass Mth 140 and I pass Mth 122 then my liberal studies math requirement will be fulfilled."

p q r

if & then

Equivalent Statements

Any two statements p and q are logically equivalent if they have the same exact meaning.

This means that p and q will always have the same truth value, in any conceivable situation.
Examples:

- "Today I have math class and today is Saturday" is equivalent to

  Today is Saturday and today I have math.

- "I have a dog or I have a cat" is equivalent to

  I have a cat or I have a dog.

Logical equivalence is denoted by the symbol $\equiv$

$p \equiv q$

The Conjunction and The Disjunction

The Conjunction

If p, q are statements, their conjunction is the statement "p and q." It is denoted by $\underbrace{p \wedge q}_{\text{p and q.}}$

Example:

Let p: "The number 2 is even." T
Let q: "The number 3 is odd" T
Then $p \wedge q$ is a new statement T
r: The number 2 is even (T) and the number 3 is odd (T).

Now consider these statements.
s: The number 1 is even (F) and the number 3 is odd (T). F
t: The number 2 is even (T) and the number 4 is odd (F). F
v : The number 3 is even (F) and the number 2 is odd (F). F

In general, in order for $p \wedge q$ to be true: both p and q have to be true.

This is summarized in the following table, called a truth table.

2 statements

2^2 → possible combinations for all of their truth values

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The Disjunction (OR notation $\vee$)

If p, q are statements, their disjunction is the statement "p or q." It is denoted: $p \vee q$ (or is inclusive)

Consider the following four statements.

q : The number 2 is even or the number 3 is odd. T T — T

r : The number 1 is even or the number 3 is odd. F T — T

s : The number 2 is even or the number 4 is odd. T F — T

t : The number 3 is even or the number 2 is odd. F F — F

In order for a disjunction to be true, at least one of the statements must be true.

The only time a disjunction is false is when all statements are false.

This is summarized in the following table, called a truth table.

→ or (disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4 rows

Basic Rules for Constructing Truth Tables

1. The number of rows depends on the number of simple statements:
   - 1 statement— 2 rows
   - 2 statements — 4 rows
   - 3 statements — 8 rows

p
T
F

T or F → $2^3 = 8$ rows

- 4 statements — 16 rows $= 2^4$

2. The number of columns depends on:

   - One column for each basic statement
   - One column for each logical connective

3. The beginning columns show every possible combination of true/false

4. Each additional column is computed from previous columns

Example 1. Make a truth table for the following statement: $(p \wedge \sim q) \vee \sim p$

p	q	$\sim q$	$C_1 \wedge C_3$ $p \wedge \sim q$	$\sim p$	$C_4 \vee C_5$ $(p \wedge \sim q) \vee \sim p$
T	T	F	F	F	F
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
C_1	C_2	C_3	C_4	C_5	C_6

Answer

Using truth tables to test for logical equivalency

To determine if two statements are equivalent, make a truth table having a column for each statement. If the columns are identical, then the statements are equivalent.

Example 2. (DeMorgan's Laws) Show these two important logical equivalencies:

1. $\sim (p \wedge q) \equiv \sim p \vee \sim q$ LHS RHS

practice 2. $\sim (p \vee q) \equiv \sim p \wedge \sim q$

p	q	$p \wedge q$	$\sim (p \wedge q)$ LHS	$\sim p$	$\sim q$	$(\sim p) \vee (\sim q)$ RHS
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

These tell us:

- The negation of "p and q" is "not p or not q"
- The negation of "p or q" is "not p and not q"

Example 3. Use DeMorgan's Laws to write the negation of each statement:

1. I want a car and a motorcycle.
2. My cat stays outside or it makes a mess.
3. I've fallen and I can't get up.
4. You study or you don't get a good grade.

1) I dont want a car or I don't want a motorcycle.
2) My cat doesn't stay outside and it doesn't make a mess.
3) I have not fallen or I can get up.
4) you don't study and you get a good grade.

Example 4. Make a truth table for $q \vee \sim(\sim p \wedge q)$.

p	q	$\sim p$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$	$q \vee \sim(\sim p \wedge q)$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	F	T	T

A **tautology** is a statement that cannot possibly be false, due to its logical structure, aka it is always true. A **contradiction** is a statement that is always false.