Math 140: Thursday, Oct 10th, Day 14 Notes

Agenda for Thursday, Oct 10th, 2024	
Chapter 3 Probability	**1**

Coming up

- ! Online HW 7 due Today
- ! Written HW 4 due Tuesday
- Office Hour **tomorrow 12-1:15pm** in C162 (not today at 1pm)
- **Exam 2** on Thursday Oct 17th - Study guide and Practice Exam posted by end of Friday
- Carrera de los Lancers - 5k this Saturday; meet in front of gym at 10am

Recall: See probability flow chart

Probabilities involving "AND" continued

Independent events revisited:

1. Consider drawing two cards (one at a time) from a deck of cards **with replacement**. Let K_{1st} be the event of "drawing a King the first time" and K_{2nd} be the event of "drawing a King the second time". Find:

formula for conditional prob

$$P(* \mid \star) = \frac{P(* \text{ and } \star)}{P(\star)}$$

$$\boxed{P(K_{2nd}) = \frac{4}{52}}$$

$$P(K_{2nd}|K_{1st}) = \frac{4}{52} = \frac{P(K_{2nd} \text{ and } K_{1st})}{P(K_{1st})} = \frac{P(K_{2nd}) \cdot \cancel{P(K_{1st})}}{\cancel{P(K_{1st})}}$$

$$* \quad \boxed{P(K_{2nd}|K_{1st}) = P(K_{2nd})}$$

when events are independent

$$P(A|B) = P(A)$$

Event A and B are independent if and only if:

- $P(A \text{ and } B) = P(A) \cdot P(B)$ ✓
- $P(A|B) = P(A)$ ✓
- $P(B|A) = P(B)$ ✓

$P(T) = \frac{1}{6}$

$P(T|F) = \frac{2}{9}$

$P(F) = \frac{3}{10}$

2. Consider drawing two cards (one at a time) from a deck **without replacement**. Let K_{1st} be the event of "drawing a King the first time" and K_{2nd} be the event of "drawing a King the second time". Find:

$$P(K_{2nd}|K_{1st}) = \frac{3}{51}$$

solve for this

can't break it down into a product

$$P(K_{2nd} \mid K_{1st}) = \frac{P(K_{1st} \text{ and } K_{2nd})}{P(K_{1st})}$$

$$P(K_{1st} \text{ and } K_{2nd}) = P(K_{1st}) \cdot P(K_{2nd} \mid K_{1st}) =$$

$$= \frac{4}{52} \cdot \frac{3}{51}$$

General Multiplication Rule

$P(* \text{ and } \star) = P(*) \cdot P(\star | *)$

The probability that two events A and B will occur in sequence is

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B|A).}$$

If events A and B are independent, then the rule can be simplified to

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B).}$$

$P(B|A) = P(B)$

!

Example 1. In a survey, 510 adults were asked if they drive a pickup truck and if they drive a Ford. The results showed that one in six adults surveyed drives a pickup truck, and three in ten adults surveyed drive a Ford. Of the adults surveyed that drive Fords, two in nine drive a pickup truck.

T

F

$P(\underset{\text{truck}}{T}) = \frac{1}{6}$, $P(F) = \frac{3}{10}$,

1. Find the probability that a randomly selected adult drives a pickup truck, **given** that the adult drives a Ford.

$$P(T|F) = \frac{2}{9} \star$$

2. Are the events "driving a Ford" and "driving a pickup truck" independent or dependent? Explain.

Check $\star$ $P(T|F) \stackrel{?}{=} P(T)$

$\frac{2}{9} \stackrel{?}{=} \frac{1}{6}$ Not independent

3. Find the probability that a randomly selected adult drives a Ford pick up truck.

$P(* \text{ and } \star) = P(*) \cdot P(\star | *)$

$P(F \text{ and } T) = P(F)P(T|F) = \frac{\cancel{3}^{1}}{10} \cdot \frac{2}{\cancel{9}_{3}} = \frac{2}{30} = \frac{1}{15}$

$P(F \text{ and } T) = \frac{2}{30}$

$P(T) = \frac{1}{6}$

$P(F) = \frac{3}{10}$

$$\frac{\cancel{3}^{\,1}}{\cancel{10}_{\,5}} \cdot \frac{\cancel{2}^{\,1}}{\cancel{9}_{\,3}} = \frac{1}{15}$$

$$\frac{3 \cdot 2}{10 \cdot 9} = \frac{6}{90} = \frac{1}{15}$$

$90 \div 6 = 15$

Using Tree Diagrams

$P(B) = .44$
$P(M) = .37$
$P(A) = .19$

The kind of picture that helps us look at conditional probabilities and multiplication rule for dependent events is called a **tree diagram**.

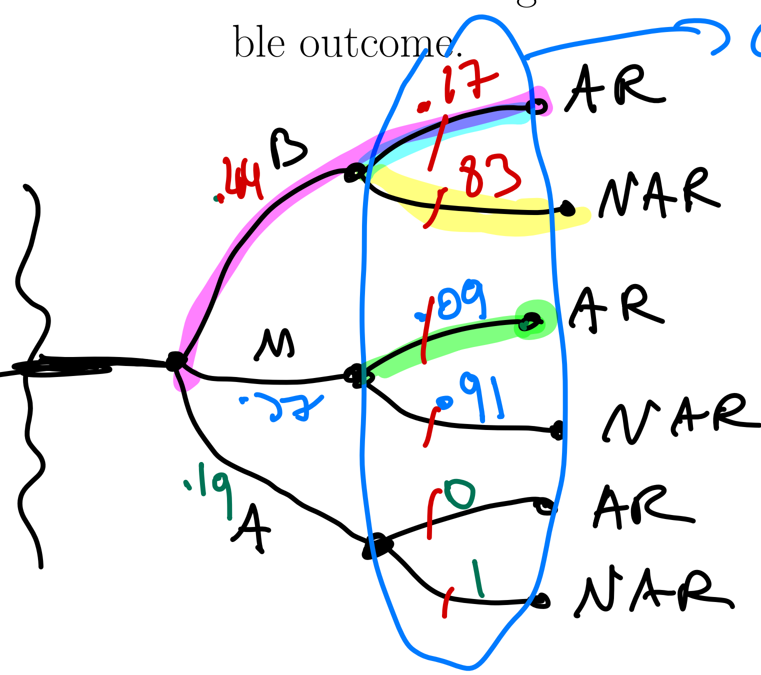
Example 2. According to a study 44% of college students engage in binge drinking, 37% drink moderately, and 19% abstain entirely. Another study finds that among binge drinkers, 17% have been involved in an alcohol-related automobile accident, while among non-bingers of the same age, only 9% have been involved in such accidents.

B — M — A

AR

NAR – not alch rel acc.

1. Build a tree diagram with corresponding probabilities on each possible outcome.



$P(AR \mid B) = .17$

$P(NAR \mid B) = .83$

2. What's the probability that a randomly selected college student will be a binge drinker **and** has had an alcohol-related car accident?

$(.44)(.17)$

3. What's the probability that a randomly selected college student will have an alcohol-related car accident **given** they are a moderate drinker?

$P(AR \mid M) = .09$

4. What's the probability that a randomly selected college student will be a moderate drinker **and** has had no alcohol-related car accident?