# Group Quiz: Integration by Parts

Your Name: ______________________________

Your Group Members' Names: ______________________________________________

**Instructions:** Work together as a group. Make sure everyone contributes to solving each problem. Show all work for full credit. I will randomly pick one quiz per group to grade, so make sure everyone agrees on the answers and everyone writes down all the work on their own paper.

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**1.** Evaluate the integral.

$$\int x^2 \cos(x)\, dx$$

**Solution:** By LIPET we let $u = x^2$, $dv = \cos(x)\, dx$, so $du = 2x\, dx$, $v = \sin(x)$.

$$\int x^2 \cos(x)\, dx = x^2 \sin(x) - \int 2x \sin(x)\, dx$$

Now we need a second IBP. Let $u = 2x$, $dv = \sin(x)\, dx$, so $du = 2\, dx$, $v = -\cos(x)$.

$$\int 2x \sin(x)\, dx = -2x\cos(x) + \int 2\cos(x)\, dx = -2x\cos(x) + 2\sin(x)$$

Putting it together:

$$\int x^2 \cos(x)\, dx = x^2 \sin(x) - \big(-2x\cos(x) + 2\sin(x)\big) + C$$

$$\boxed{\int x^2 \cos(x)\, dx = x^2 \sin(x) + 2x\cos(x) - 2\sin(x) + C}$$

**2.** Evaluate the integral.

$$\int \frac{\ln(x)}{x^2}\, dx$$

**Solution:** By LIPET we let $u = \ln(x)$, $dv = x^{-2}\, dx$, so $du = \frac{1}{x}\, dx$, $v = -\frac{1}{x}$.

$$\int \frac{\ln(x)}{x^2}\, dx = -\frac{\ln(x)}{x} - \int \left(-\frac{1}{x}\right)\left(\frac{1}{x}\right) dx = -\frac{\ln(x)}{x} + \int \frac{1}{x^2}\, dx$$

$$= -\frac{\ln(x)}{x} - \frac{1}{x} + C$$

$$\boxed{\int \frac{\ln(x)}{x^2}\, dx = -\frac{\ln(x)}{x} - \frac{1}{x} + C}$$

**3.** Evaluate the definite integral.

$$\int_0^{4\pi} t^2 \sin(2t)\, dt$$

**Solution:** By LIPET we let $u = t^2$, $dv = \sin(2t)\, dt$, so $du = 2t\, dt$, $v = -\frac{1}{2}\cos(2t)$.

$$\int t^2 \sin(2t)\, dt = -\frac{t^2}{2}\cos(2t) + \int t\cos(2t)\, dt$$

Now we need IBP again for

$$\int t\cos(2t)\, dt$$

By LIPET we let $u = t$, $dv = \cos(2t)\, dt$, so $du = dt$, $v = \frac{1}{2}\sin(2t)$.

$$\int t\cos(2t)\, dt = \frac{t}{2}\sin(2t) - \int \frac{1}{2}\sin(2t)\, dt = \frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t)$$

So the antiderivative is

$$F(t) = -\frac{t^2}{2}\cos(2t) + \frac{t}{2}\sin(2t) + \frac{1}{4}\cos(2t)$$

Evaluate at the bounds. At $t = 4\pi$: $\cos(8\pi) = 1$, $\sin(8\pi) = 0$, so

$$F(4\pi) = -\frac{16\pi^2}{2}(1) + \frac{4\pi}{2}(0) + \frac{1}{4}(1) = -8\pi^2 + \frac{1}{4}$$

At $t = 0$: $\cos(0) = 1$, $\sin(0) = 0$, so

$$F(0) = 0 + 0 + \frac{1}{4} = \frac{1}{4}$$

Therefore

$$\boxed{\int_0^{4\pi} t^2 \sin(2t)\, dt = F(4\pi) - F(0) = -8\pi^2}$$

Bonus: Evaluate the integral.

$$\int e^{\cos t}\sin(2t)\, dt$$

Hint: You will need to use a trig identity first then use a u-sub.
**Solution:** Use the double angle identity $\sin(2t) = 2\sin(t)\cos(t)$:

$$\int e^{\cos t}\sin(2t)\, dt = \int e^{\cos t}\cdot 2\sin(t)\cos(t)\, dt = 2\int \cos(t)\, e^{\cos t}\sin(t)\, dt$$

We do an x-sub instead of a u-sub so we don't get confused with our u's for IBP. Let $x = \cos(t)$, so $dx = -\sin(t)\, dt$, i.e. $\sin(t)\, dt = -dx$:

$$2\int x\, e^x(-dx) = -2\int x\, e^x\, dx$$

We do integration by parts on $\int x\,e^x\,dx$: let $u = x$, $dv = e^x\,dx$, so $du = dx$, $v = e^x$:

$$\int x\,e^x\,dx = x\,e^x - \int e^x\,dx = x\,e^x - e^x$$

So

$$-2\int x\,e^x\,dx = -2\big(x\,e^x - e^x\big) + C = -2x\,e^x + 2e^x + C$$

Substituting back $x = \cos(t)$:

$$\boxed{\int e^{\cos t}\sin(2t)\,dt = 2e^{\cos t}\big(1 - \cos(t)\big) + C}$$