$2x + 1 = 5$
$\quad -1 \quad -1$
$2x = 4$

$x = 2$

# Solving Trigonometric Equations

The goal of solving trigonometric equations is to isolate the trigonometric function involved in the equation by using standard algebraic techniques. Once isolated, we can apply inverse trigonometric functions or use known values to determine the solution. We must also consider the periodic nature of trigonometric functions when identifying all possible solutions within a given interval.

## Example 1 Solving a Trigonometric Equation

Solve $2\sin x - 1 = 0$.

**Solution:**

$2\sin x = 1$

$\sin x = \frac{1}{2}$

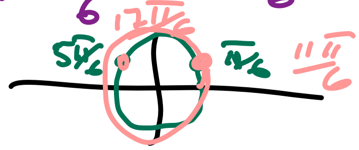
$x = \frac{\pi}{6}$

$x = \frac{5\pi}{6}$

solution in $[0, 2\pi)$

$n = 1$

$x = \frac{\pi}{6} + 2\pi = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6}$




All solutions $x = \frac{\pi}{6} + 2n\pi$

$x = \frac{5\pi}{6} + 2n\pi$

general solution

$2x + 3 = x - 5$

## Example 2 Collecting Like Terms

Find all solutions of

$$\sin x + \sqrt{2} = -\sin x$$

$+\sin x \qquad +\sin x$

in the interval $[0, 2\pi)$.

**Solution:**

$2\sin x + \sqrt{2} = 0$

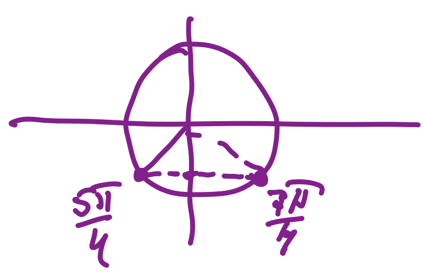
$2\sin x = -\sqrt{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

$\sin x = ?$ | $x = ?$

$x = \frac{5\pi}{4}$

$x = \frac{7\pi}{4}$



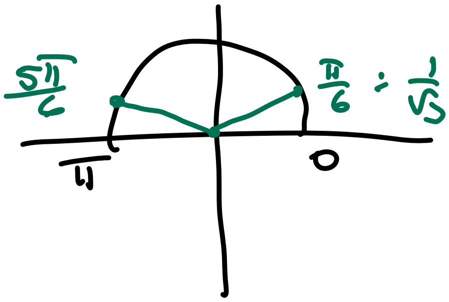
## Example 3 Extracting Square Roots

Solve $3\tan^2 x - 1 = 0$.

**Solution:**

$$3\tan^2 x = 1$$
$$\tan^2 x = 1/3$$
$$\tan x = \pm\sqrt{\tfrac{1}{3}}$$
$$\tan = \pm\frac{1}{\sqrt{3}}$$

period of tang is $\pi$




General solution:

$$x = \pi/6 + n\pi$$
$$x = \frac{5\pi}{6} + n\pi$$

## Example 4 Factoring

Solve $\cot x \cos^2 x = 2\cot x$.

**Solution:**

$$\frac{1}{2}\cot x \cos^2 x = \cot x$$
$$-\cot x \qquad -\cot x$$
$$\frac{1}{2}\cot x\cos^2 x - \cot x = 0$$
$$\underbrace{\cot x}_{=0}\underbrace{\left(\frac{1}{2}\cos^2 x - 1\right)}_{=0} = 0$$
$$\frac{1}{2}\cos^2 x - 1 = 0 \Rightarrow \cos x = \pm\sqrt{2} \rightarrow \text{no solutions} \quad \text{range of cos } [-1,1]$$

$\cot x = 0$

~~$\pi, 0$~~

~~$\cot x\ 0 = \frac{\cos 0}{\sin 0} = \frac{1}{0}$~~ undefin


$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$

Solution $x = \frac{\pi}{2} + n\pi$

## Example 5 Factoring an Equation of Quadratic Type

Find all solutions of $2\sin^2 x - \sin x - 1 = 0$ in the interval $[0, 2\pi)$.

**Solution:**

Let $\sin x = y$

$$2y^2 - y - 1 = 0$$
$$\underbrace{(2y+1)}_{=0}\underbrace{(y-1)}_{=0} = 0$$

$2y + 1 = 0$

$2y = -1$

$y = -1/2$

$\sin x = -1/2$

$x = \frac{7\pi}{6}$

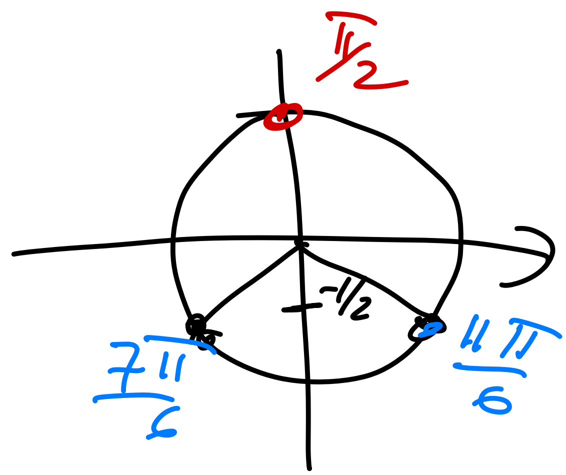
$x = \frac{11\pi}{6}$

$y - 1 = 0$

$y = 1$

$\sin x = 1$

$x = \frac{\pi}{2}$



## Example 6 Rewriting with a Single Trigonometric Function

To solve $2\sin^2 x + 3\cos x - 3 = 0$, begin by rewriting the equation so that it has only cosine functions.
**Solution:**

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$2(1-\cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$-2\cos^2 x + 3\cos x - 1 = 0 \quad / (-1)$

$2\cos^2 x - 3\cos x + 1 = 0$ $\qquad \cos x = u$

$2u^2 - 3u + 1 = 0$

$(\underbrace{2u-1}_{=0})(\underbrace{u-1}_{=0}) = 0$

HW
Look up AC method for factoring quadratic.

$2u - 1 = 0$

$u = 1/2$

$\cos x = 1/2$

$x = \pi/3 + 2n\pi$

$x = 5\pi/3 + 2n\pi$

$u - 1 = 0$

$u = 1$

$\cos x = 1$

$x = 0 + 2n\pi$

$x = 2n\pi$

## Example 7 Squaring and Converting to Quadratic Type

Find all solutions of $\cos x + 1 = \sin x$ in the interval $[0, 2\pi)$.
**Solution:**

$(\cos x + 1)^2 = (\sin x)^2 \quad |$ square both !!!!

check solutions at the end

$\cos^2 x + 2\cos x + 1 = \underbrace{\sin^2 x}_{1-\cos^2 x}$

$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$

$+\cos^2 x \quad -1 \qquad -1 \quad +\cos^2 x$

$2\cos^2 x + 2\cos x = 0$

$\cos x(2\cos x + 2) = 0$

$2\cos x(\cos x + 1) = 0$

$\cos x = 0$

$x = \frac{\pi}{2}$

$\cancel{x = \frac{3\pi}{2}}$

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

check

$x = \frac{\pi}{2}$ ✓

$\boxed{\cos\frac{\pi}{2} + 1 = \sin\frac{\pi}{2}}$

$0 + 1 = 1$

$\cancel{x = \frac{3\pi}{2}}$

$\cos\frac{3\pi}{2} + 1 \stackrel{?}{=} \sin\frac{3\pi}{2}$

$0 + 1 = -1$

$x = \pi$ ✓

$\cos\pi + 1 \stackrel{?}{=} \sin\pi$

$-1 + 1 = 0$

## Example 8 Functions of Multiple Angles

Solve $2\cos 3t - 1 = 0$. $t = ?$

**Solution:**

$2\cos 3t = 1$

$\cos(3t) = 1/2$

$\boxed{3t = \frac{\pi}{3} + 2n\pi}$ $/\div 3 \Rightarrow$

$3t = \frac{5\pi}{3} + 2n\pi$ $/\div 3 \Rightarrow$

$$\boxed{\begin{aligned} t &= \frac{\pi}{9} + \frac{2n\pi}{3} \\ t &= \frac{5\pi}{9} + \frac{2n\pi}{3} \end{aligned}}$$

## Example 9 Functions of Multiple Angles

Solve $3\tan\left(\frac{x}{2}\right) + 3 = 0$ $x = ?$

**Solution:**

$3\tan\left(\frac{x}{2}\right) = -3$

$\tan\left(\frac{x}{2}\right) = -1$

$\frac{x}{2} = \frac{3\pi}{4} + n\pi$ $/\cdot 2$

$$\boxed{x = \frac{3\pi}{2} + 2n\pi}$$

## Example 10 Using Inverse Functions

Solve $\sec^2 x - 2\tan x = 4$. $1 + \tan^2 x = \sec^2 x$

**Solution:**

$1 + \tan^2 x - 2\tan x - 4 = 0$

$\tan^2 x - 2\tan x - 3 = 0$

$\tan x = u$

$u^2 - 2u - 3 = 0$

$(\underbrace{u-3}_{=0})(\underbrace{u+1}_{=0}) = 0$

$u - 3 = 0$

$u = 3$

$\tan x = 3$

$\arctan(\tan x) = \arctan 3$

$x = \arctan 3 + n\pi$

$u + 1 = 0$

$u = -1$

$\tan x = -1$

$$\boxed{x = \frac{3\pi}{4} + n\pi}$$