Math 140: Chapter 3 Probability

Recall: See probability flow chart

Probabilities involving "AND"

Independent events are events that don't depend on each other. The occurrence of one of the events does not affect the occurrence of the other.

Example 1. Are the following events independent?

1. Probability of your "flight being on time on Monday 8am" is $P(F) = .85$. Are the events "flight being on time this Monday" and "flight being on time the following Monday" independent?

   yes independent events.

2. Consider drawing two cards (one at a time) from a deck of cards **with replacement**. Let A be the event of "drawing a King the first time" and B be the event of "drawing a King the second time".

   yes independent!

3. Consider drawing two cards (one at a time) from a deck **without replacement**. Let A be the event of "drawing a King the first time" and B be the event of "drawing a King the second time".

   NOT independent (I have 51 card for second draw)

Multiplication Rule for independent events

For two independent events A and B , the probability that both A and B occur is the product of the probabilities of the two events.

$$\underbrace{P(A \text{ and } B)}_{\text{in seq.}} = P(A) \cdot P(B)$$

Example 2. When you get to the light at W Washington St and Lancer Ln, the light is red, green, or yellow. Suppose we know that probability of green light is $P(G) = 0.35$ and $P(Y) = 0.04$.

1. What is the probability of red?

$P(G) + P(Y) + P(R) = 1$; $0.35 + 0.04 + P(R) = 1$

$P(R) = 1 - (0.39) = .61$

2. What is the probability that on your morning commute you find the light red both Monday and Tuesday?

$P(\underset{\text{first on Mond}}{R} \text{ and } \underset{\text{on Tuesd}}{R}) = P(R) \cdot P(R) = (0.61)(0.61) = 0.093$

0.372

3. What is the probability of not encountering a red light?

$P(\text{not } R) = 1 - P(R) = 1 - 0.61 = 0.39$

4. What is the probability that on your morning commute you don't encounter a red light until Wednesday?

$P(\underset{M}{\text{not } R} \text{ and } \underset{T}{\text{not } R} \text{ and } \underset{W}{R}) = P(\text{not } R) \cdot P(\text{not } R) \cdot P(R)$

$= (0.39)(0.39)(0.61)$

$= 0.093$

Example 3. Two psychologists surveyed 478 children in elementary schools in Michigan. Among other questions, they asked the students whether their primary goal was to get good grades, to be popular, or to be good at sports.

	Grades	Popular	Sports	Total
Boy	117	50	60	227
Girl	130	91	30	251
Total	247	141	90	478

1. If we select a student at random from this study, what is the probability we select a girl ?

   P(girl)= $\frac{n(\text{girls})}{\text{total}} = \frac{251}{478} = 0.525$

   simple event

2. What's the probability of selecting a girl whose goal is to be be good at sports?

   P(girl and Sports)= $\frac{30}{478} = 0.063 \neq P(\text{girl}) \cdot P(\text{sports})$

   one person

3. What's the probability of first selecting a girl and second selecting a person whose goal is to be be good at sports? (with replacement)

   P(first girl and second Sports)= multiplying probabilities

   $= P(\text{girl}) \cdot P(\text{sports}) = \frac{251}{478} \cdot \frac{90}{478} = 0.09$

   sequence

4. What's the probability of selecting any student whose goal is to excel at sports?

   P(sports)= $\frac{90}{478} = 0.188$

5. What if we are **given** the information that the selected student is a girl? Would that change the probability that the selected student's goal is sports?

$P(\text{sports given girl}) = P(\text{sports} \mid \text{girl}) = \frac{30}{251}$ (given)

$= 0.120$

Conditional probability

A probability that takes into account a given condition such as this is called a **conditional probability**.

$$\boxed{P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}}$$

$P(\text{sports} \mid \text{girl}) = \frac{P(\text{sport and girl})}{P(\text{girl})}$

$= \frac{30/\cancel{478}}{251/\cancel{478}} = \frac{30}{251}$

Example 4. Our survey found that 33% of college students are in a relationship, 25% are involved in sports, and 11% are both.

$P(R) = 0.33$, $P(S) = 0.25$, $\boxed{P(R \text{ and } S) = .11}$

1. While dining in a campus facility open only to students, you meet a student who is on a sports team. What is the probability that your new acquaintance is in a relationship?

given

$P(R \text{ given } S) = P(R \mid S) = \frac{P(R \text{ and } S)}{P(S)} = \frac{.11}{0.25} = 0.44$

2. While dining in a campus facility open only to students, you meet a student who is in a relationship. What is the probability that your new acquaintance is involved in sports?

given

$P(S \text{ given } R) = P(S \mid R) = \frac{P(S \text{ and } R)}{P(R)} = \frac{.11}{0.33}$

$= .333$

Caution:

!!! $\boxed{P(A \mid B) \neq P(B \mid A)}$