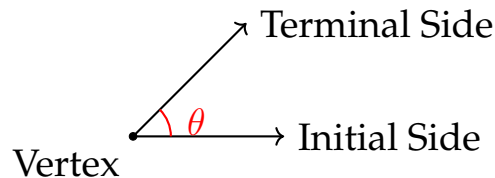


1 Angles

1.1 Basic Definitions

Definition : Angle

An **angle** is formed by two rays that share a common endpoint. The common endpoint is called the **vertex** of the angle, one ray is called the **initial side**, and the other ray is called the **terminal side**.



Definition : Standard Position

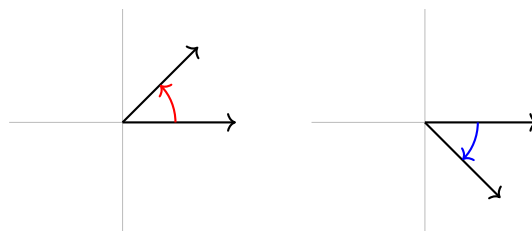
An angle is in **standard position** if:

- The vertex is at the origin of a rectangular coordinate system
- The initial side is along the positive x -axis

Notation: we use Greek letters to denote angles

Definition : Positive and Negative Angles

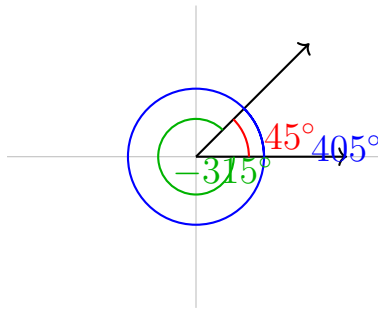
- A **positive angle** is generated by a _____ rotation from the initial side to the terminal side.
- A **negative angle** is generated by a _____ rotation from the initial side to the terminal side.



Definition : Coterminal Angles

Coterminal angles are angles in standard position that have _____.

If θ is an angle, then $\theta + 360^\circ n$ where n is an integer, is coterminal with θ .



Coterminal Angles: 45° , 405° , and -315°

Example 1: Finding Coterminal Angles

Find two coterminal angles (one positive and one negative) for $\theta = 60^\circ$.

Solution:

For a positive coterminal angle, add 360° :

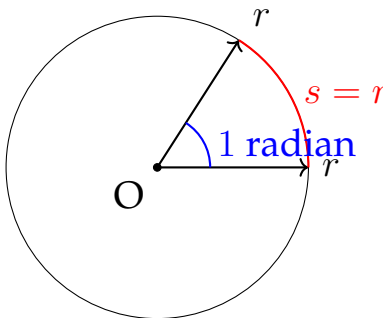
For a negative coterminal angle, subtract 360° :

2 Radian Measure

2.1 Definition of Radian

Definition : Radian

A **radian** is the measure of an angle that, when placed at the center of a circle, intercepts an arc equal in length to the radius of the circle.



Definition : Central Angle

A **central angle** is an angle whose vertex is at the center of a circle, with its sides being radii of the circle.

3 Degree and Radian Conversions

3.1 Converting Between Degrees and Radians

Definition : Conversion Formulas

To convert between degrees and radians, use the following relationships:

$$1 \text{ radian} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$
$$1^\circ = \frac{\pi}{180} \text{ radians} \approx 0.01745 \text{ radians}$$

Therefore:

$$\text{radians} = \text{degrees} \times \frac{\pi}{180}$$
$$\text{degrees} = \text{radians} \times \frac{180}{\pi}$$

Example 2: Converting from Degrees to Radians

Convert the following angles from degrees to radians in terms of π : a) 45° b) 120°
c) -30°

Solution:

a) 45° in radians:

b) 120° in radians:

c) -30° in radians:

Example 3: Converting from Radians to Degrees

Convert the following angles from radians to degrees: a) $\frac{\pi}{3}$ b) $\frac{5\pi}{4}$ c) $-\frac{\pi}{2}$

Solution:

a) $\frac{\pi}{3}$ in degrees:

b) $\frac{5\pi}{4}$ in degrees:

c) $-\frac{\pi}{2}$ in degrees:

4 Complementary, Supplementary Angles, and Arc Length

4.1 Complementary and Supplementary Angles

Definition : Complementary Angles

Two angles are **complementary** if their sum equals 90° or $\frac{\pi}{2}$ radians.

If angles A and B are complementary, then:

$$A + B = 90^\circ \text{ or } A + B = \frac{\pi}{2} \text{ radians}$$

The **complement** of angle A is $(90^\circ - A)$ or $(\frac{\pi}{2} - A)$ in radians.

Definition : Supplementary Angles

Two angles are **supplementary** if their sum equals 180° or π radians.

If angles A and B are supplementary, then:

$$A + B = 180^\circ \text{ or } A + B = \pi \text{ radians}$$

The **supplement** of angle A is $(180^\circ - A)$ or $(\pi - A)$ in radians.

Example 4: Finding Complements and Supplements

Find the complement and supplement of each angle: a) 25° , b) $\frac{\pi}{4}$ radians, and c) 100°

Solution:

a) For $\theta = 25^\circ$:

Complement =

Supplement =

b) For $\theta = \frac{\pi}{4}$ radians:

Complement =

Supplement =

c) For $\theta = 100^\circ$:

Complement =

Supplement =

4.2 Arc Length and Area of a Sector

Definition : Arc Length and Area of a sector

If a central angle θ in a circle of radius r intercepts an arc, then the arc length s is given by:

$$s = r\theta$$

where θ must be measured in radians. The area A of a sector is given by:

$$A = \frac{1}{2}r^2\theta$$

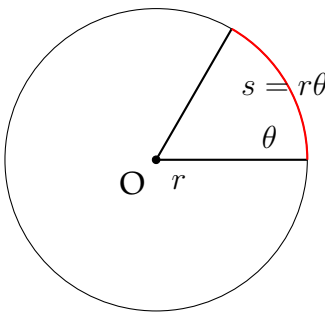


Figure 1: Arc length of a sector

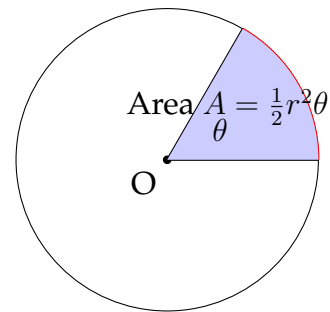


Figure 2: Area of a sector

Example 5: Finding Arc Length

Find the arc length intercepted by a central angle of $\frac{\pi}{3}$ radians in a circle with radius 12 cm.

Solution: Using the arc length formula with $r = 12$ cm and $\theta = \frac{\pi}{3}$ radians:

Example 6: Finding Area of a Sector

Find the area of a sector formed by a central angle of 72° in a circle with radius 5 m.

5 The Unit Circle and Trigonometric Functions

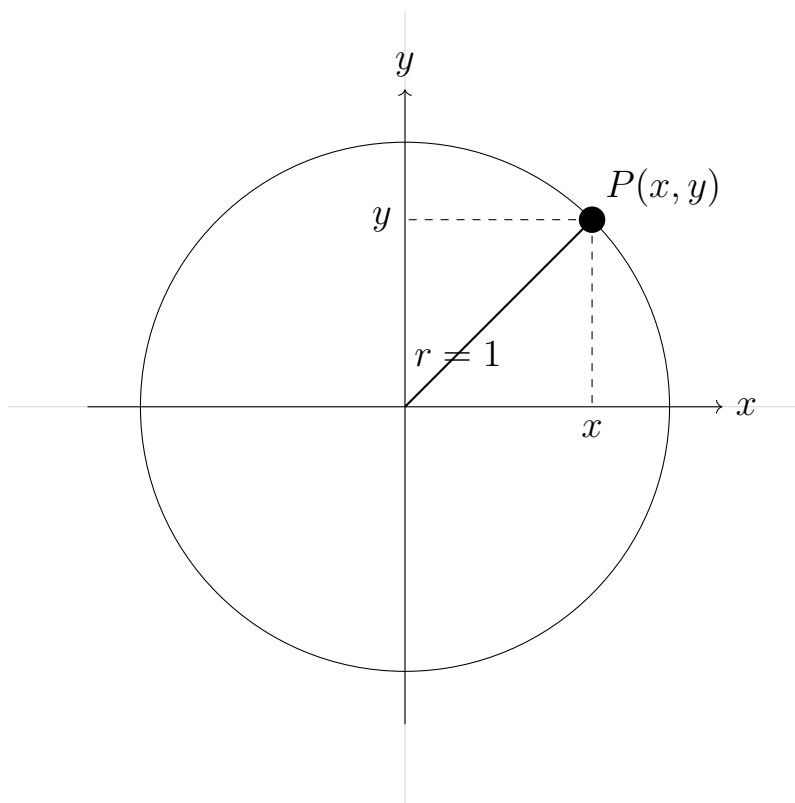
5.1 Definition of the Unit Circle

Definition : Unit Circle

The **unit circle** is a circle with radius 1 and center at the origin of the coordinate plane. Every point on the unit circle is at a distance of 1 unit from the origin.

The equation of the unit circle is:

$$x^2 + y^2 = 1$$



5.2 Six Trigonometric Functions: sine, cosine, tangent, cosecant, secant, cotangent

Definition : Primary Trigonometric Functions

For a point $P(x, y)$ on the unit circle corresponding to an angle θ in standard position:

$$\sin \theta =$$

$$\cos \theta =$$

$$\tan \theta = \quad , \quad \cos \theta \neq 0$$

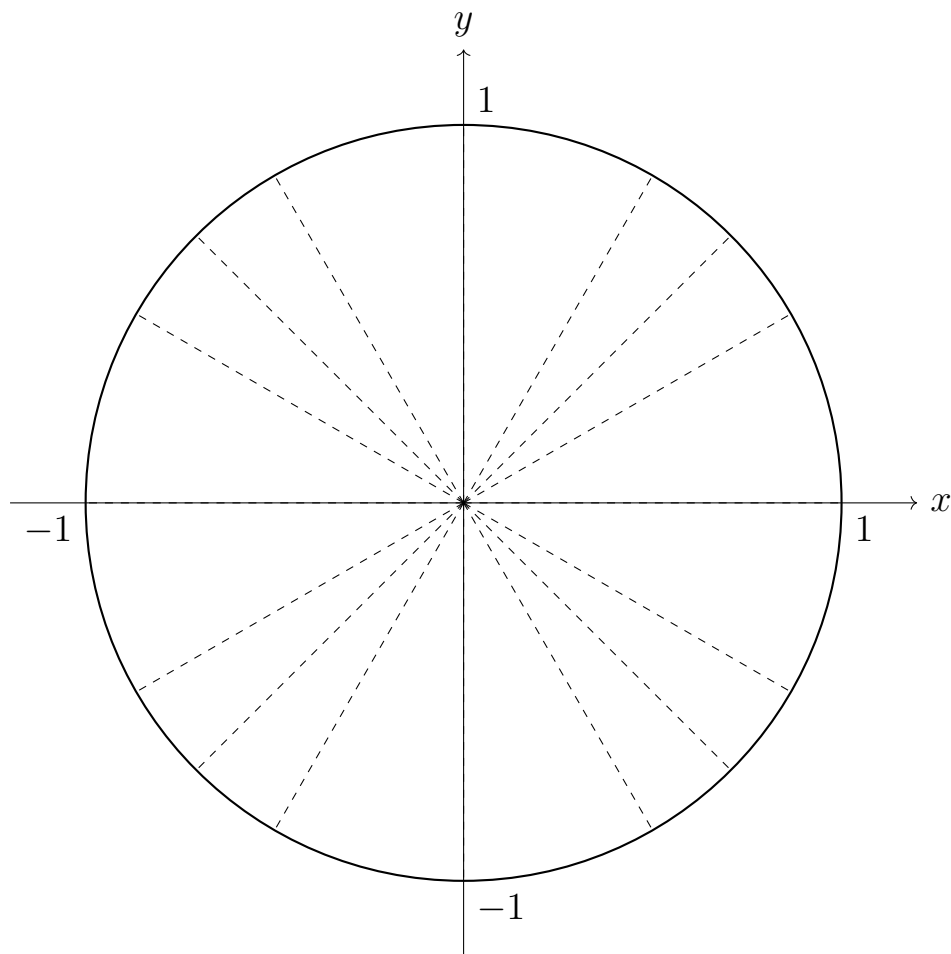
Definition : Reciprocal Trigonometric Functions

The reciprocal trigonometric functions are defined as:

$$\csc \theta = \quad , \quad \sin \theta \neq 0$$

$$\sec \theta = \quad , \quad \cos \theta \neq 0$$

$$\cot \theta = \quad , \quad \sin \theta \neq 0$$



Unit Circle with Common Angles

5.3 Evaluating Trigonometric Functions

Example 7: Evaluating Trig Functions for Special Angles

Find the values of all six trigonometric functions for $\theta = 30^\circ$.

Solution:

For $\theta = 30^\circ = \frac{\pi}{6}$ radians, the point on the unit circle is:

$$P(\cos 30^\circ, \sin 30^\circ) = P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

Therefore:

$$\sin 30^\circ =$$

$$\cos 30^\circ =$$

$$\tan 30^\circ =$$

$$\csc 30^\circ =$$

$$\sec 30^\circ =$$

$$\cot 30^\circ =$$

Example 8: Finding Trig Functions from a Point

If $P(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ is a point on the unit circle corresponding to angle θ , find all six trigonometric functions of θ .

Solution:

Since the point is on the unit circle, we know that:

$$\cos \theta =$$

$$\sin \theta =$$

The other trigonometric functions can be found using these values:

$$\tan \theta =$$

$$\csc \theta =$$

$$\sec \theta =$$

$$\cot \theta =$$

5.4 Domains of Trigonometric Functions

Definition : Domains of Trigonometric Functions

The domains of the six trigonometric functions are:

$\sin \theta$: All real numbers

$\cos \theta$: All real numbers

$\tan \theta$: All real numbers except $\theta = \frac{\pi}{2} + n\pi$ (where n is an integer)

$\csc \theta$: All real numbers except $\theta = n\pi$ (where n is an integer)

$\sec \theta$: All real numbers except $\theta = \frac{\pi}{2} + n\pi$ (where n is an integer)

$\cot \theta$: All real numbers except $\theta = n\pi$ (where n is an integer)

5.5 Even and Odd Trigonometric Functions

Definition : Even and Odd Functions

A function f is **even** if $f(-x) = f(x)$ for all x in the domain. A function f is **odd** if $f(-x) = -f(x)$ for all x in the domain.

Note : Even and Odd Trigonometric Functions

5.6 Periodicity of Trigonometric Functions

Definition : Periodic Functions

A function f is **periodic** if there exists a positive real number p such that $f(x+p) = f(x)$ for all x in the domain. The smallest such value of p is called the **period** of the function.

Note : Periods of Trigonometric Functions

Example 9: Using Even/Odd Properties

Find the exact value of $\cos(-5\pi/4)$ without using a calculator.

Solution:

6 Summary of Key Concepts

1. **Angles** are formed by two rays with a common vertex; they have an initial side and a terminal side.
2. **Standard position** means the angle's vertex is at the origin and the initial side is along the positive x-axis.
3. **Positive angles** rotate counterclockwise; **negative angles** rotate clockwise.
4. **Coterminal angles** share the same terminal side and differ by multiples of 360° or 2π radians.
5. **Radians** and **degrees** are two common angle measures: π radians $= 180^\circ$.
6. **Complementary angles** sum to 90° or $\frac{\pi}{2}$ radians.
7. **Supplementary angles** sum to 180° or π radians.
8. **Arc length** formula: $s = r\theta$ (where θ is in radians)
9. **Area of a sector** formula: $A = \frac{1}{2}r^2\theta$ (where θ is in radians)
10. **Unit circle** has radius 1 and is centered at the origin. Points on the unit circle are $(x, y) = (\cos \theta, \sin \theta)$.
11. **Six trigonometric functions**: sine, cosine, tangent, cosecant, secant, and cotangent.
12. **Even functions**: $\cos \theta$ and $\sec \theta$
13. **Odd functions**: $\sin \theta$, $\csc \theta$, $\tan \theta$, and $\cot \theta$
14. **Periodicity**: $\sin \theta$, $\cos \theta$, $\csc \theta$, and $\sec \theta$ have period 2π ; $\tan \theta$ and $\cot \theta$ have period π .