

We have one more type of asymptote for rational functions.

Let $f(x) = \frac{P(x)}{Q(x)}$ be a rational function. If the degree of the top polynomial $P(x)$ is exactly one more than the degree of the bottom polynomial $Q(x)$, then the function $f(x)$ will have a slant or oblique asymptote of the form $y = mx + b$.

Example 1. How to get the slant asymptote. Let $f(x) = \frac{x^3}{2x^2 - 2} = \frac{x^3}{2(x-1)(x+1)}$

you do long division

divisor $\rightarrow$ $2x^2 - 2$

$$\begin{array}{r} \frac{1}{2}x \\ 2x^2 - 2 \overline{) x^3 + 0x^2 + 0x + 0} \\ \underline{x^3} \\ -x \\ \underline{-x} \\ x \end{array}$$

quotient $\rightarrow$ $\frac{1}{2}x$ (slant asym.)

dividend $\rightarrow$ x (remainder)

$\frac{x^3}{2x^2 - 2} = \frac{1}{2}x + \frac{x}{2x^2 - 2}$

$\downarrow$
 slant asymptote

$\frac{x^3}{2x^2} = \frac{1}{2}x$
 $\frac{1}{2}x \cdot 2x^2$
 $\frac{1}{2}x(-2)$