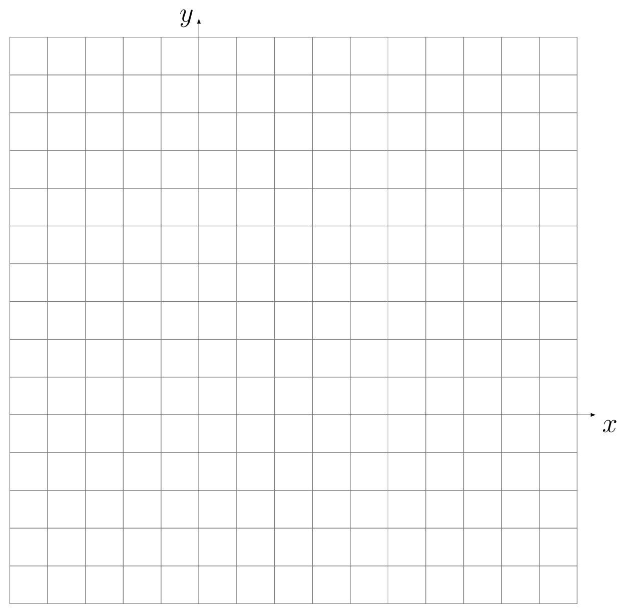
Review - Logarithmic Functions an Laws of Logs

1. Introduction

Graph the exponential function with base 2 and initial value of 1, $f(x) = 2^x$. Let's say that this function computes the number of bacteria after x hours. Use a table of values with at least 4 points to draw the graph. Is this function one-to-one? On the same graph, draw its inverse.



x				
$f(x)$				

x				
$f^{-1}(x)$				

What is the input for the inverse?

What is the output for the inverse?

The inverse of the exponential function with base 2 is called the **logarithmic function** with base 2 and it is written as $f(x) = \log_2 x$.

2. Evaluate logarithms

Use the properties of logarithms to find the exact values of the expressions.

1. $\log_2 32 =$
2. $\log_2 \dfrac{1}{2} =$
3. $\log_{10} 100 =$
4. $\log_{10} 1 =$
5. $\log_{10} 0.001 =$
6. $\log_e e =$

When the base of the logarithm is 10 we call that logarithm the **common logarithm** and we do not write the base 10.
So from now on, if the base is not specified, it means it is base 10. For example $\log 1000 = 3$.

When the base of the logarithm is e we call that logarithm the **natural logarithm** and instead of $\log_e z$ we write $\ln z$.
So from now on if you're seeing $\ln$ of something, it means the base is e. For example $\ln e = 1$.

3. Graphs of logarithmic functions

We will mostly use the common and natural logarithm. To graph them you use you calculator and the $\log$ and $\ln$ buttons.

So for example if you're asked to graph $f(x) = \log(x)$ simply put that in y_1 and adjust your window properly. What is the domain and what is the range?

4. Properties of logarithms

Just like we have rules for exponents we will also have rules for logarithms. Let's recall some rules for exponents.

1. $a^0 = 1$

   What would that mean in terms of logarithms?

2. $a^1 = a$

   What would that mean in terms of logarithms?

3. Since exponential functions and logarithmic functions are inverses of each other, what happens if I take the input x and I apply the exponential function and then apply the logarithmic function with the same base, or vice versa?

   $a^{\log_a x} = x$

   $\log_a a^x = x$

4. Recall that when we multiply two exponential with the same base we add the exponents. For example $2^x 2^y = 2^{(x+y)}$. Similarly for logarithms we have the following properties:

   $\log_a(mn) = \log_a m + \log_a n$

   $\log_a \frac{m}{n} = \log_a m - \log_a n$

5. The most important property of logarithms is this one.

   $\log_a m^x = x \log_a m$

This property allows us to bring the exponent down in front of the logarithm and it's very useful when we want to solve a problem where the unknown is an exponent.

Recall the problem from the previous lecture:

After t days, the amount of Thorium-234 in a sample is $a(t) = 45e^{-0.029t}$ micrograms.

We were asked to find when the amount halves; so our unknown is t and t appears as an exponent. We are going to have to use logarithms to solve this, more specifically the above property of logarithms.

Practice: Use the properties of logarithms to expand the following expressions:

1. $\ln 17x^2 =$

2. $\log \dfrac{7}{x} =$

3. $\ln \dfrac{x^4}{y^3\sqrt{z^7}} =$

Practice: Use the properties of logarithms to rewrite as a single logarithm the following expressions:

1. $\log 22 + \log \dfrac{1}{11} =$

2. $\log x - 2\log y + \dfrac{1}{2}\log z =$

5. Logarithmic and exponential equations

All the word problems that involve exponential function and logarithmic functions are going to require for us to know how to solve exponential and logarithmic equations.

Solve the following equations. Keep in mind that exponential functions and logarithmic functions are inverses of each other.

1. $\ln e^{-x} = 21$

2. $10^x - 2 = 998$

3. $5(10^{4y}) = 31$

4. $\log x = 4$

5. $9 \log x = 18$

6. Applications

Use what you have learned to solve the following applied problems. Notice that generally, an applied problem will require a logarithm when you have an exponential model and you are asked to find the time at which a given amount occurs.

1. A pot of boiling water with a temperature of $100°$C is set in a room. The temperature T of the water after x hours is given by $T(x) = 20 + 80e^{-x}$.

   (a) Estimate the temperature of the water after 2 hours.

   (b) How long did it take for the water to cool to $30°$C?

2. The number of fish in thousands x years after 1975 can be modeled by $f(x) = 221(0.873)^x$.

   (a) Estimate the year when the number of these fish reached 93 thousands.

   (b) Estimate the year when the number of fish halved.

   (c) Estimate the year when the number of fish decreased to 30% of the initial population.

3. A substance currently consists of C grams and it decays at a continuous rate of 10.2% per year.

   (a) Create a model to represent the amount A, of substance remaining after x years.

   (b) Determine the number of years it will take for the amount of the substance to decrease to 10% of its current amount.

   (c) After 17 years there was approximately 407 grams of the substance remaining. To the nearest gram, how many grams of this substance were initially present?