R5 in textbook

Day 15 Part 2: Factoring Techniques

1. Factoring out the Greatest Common Factor (GCF)

Key Concept: The first step in factoring any expression is to check whether all terms share a common factor.

12: 1, 2, 3, 4, 6, 12
18: 1, 2, 3, 6, 9, 18
24: 1, 2, 3, 4, 6, 8, 12, 24

Process:

1. Identify the GCF of all terms in the expression
2. Factor out the GCF from each term
3. Write as GCF $\times$ (remaining factors)

Example 1: Factor $12x^5 + 18x^4 - 24x^3$ $= 6x^3(2x^2+3x-4)$
Step 1: Find the GCF of the terms.
GCF of coefficients: GCF of 12, 18, and 24 = 6
GCF of variables: GCF of x^5, x^4, and x^3 = x^3
Therefore, the GCF is $6x^3$
Step 2: Factor out the GCF from each term.
$12x^5 + 18x^4 - 24x^3 = 6x^3(\;2x^2\; + \;3x\; - \;4\;)$

$\frac{12x^5}{6x^3} = 2x^{5-3} = 2x^2$

Example 2: Factor $3y(2y-5) + (2y-5)$
Step 1: Recognize that $(2y-5)$ appears in both terms.
Therefore, the GCF is $(2y-5)$
Step 2: Factor out the GCF from each term.
$3y(2y-5) + (2y-5) = (2y-5)(\;3y\; + \;1\;)$

$(2y-5)(3y+1)$

Example 3: Factor $-4x^3 - 12x^2 + 8$
Step 1: Notice that we can factor out -4 as a negative GCF.
GCF = -4
Step 2: Factor out the negative GCF.
$-4x^3 - 12x^2 + 8 = -4(\;x^3\; + \;3x^2\; - \;2\;)$

Practice

Factor each expression completely:

a) $5x^4 - 15x^3 + 10x^2$

b) $-7y^3 + 21y^2 - 14y$

c) $6z(z+4) - 2(z+4)$

2. Factoring by Grouping

Key Concept: When an expression has four terms, we can sometimes factor it by grouping terms in pairs.

Process:

1. Group the first two terms and last two terms
2. Factor out the GCF from each group
3. If the expressions in parentheses are identical, factor out this common binomial

Example 1: Factor $2ax - 6ay + 5x - 15y$

$= (2ax - 6ay) + (5x - 15y)$

$= 2a(x - 3y) + 5(x - 3y)$

$= (x-3y)(2a + 5)$

1 3

Example 2: Factor $m^2 - 3n + 3m - mn =$

$$= (m^2 + 3m) - 3n - mn$$
$$= (m^2 + 3m) - (3n + mn)$$
$$= m(m+3) - n(3+m)$$
$$= (m+3)(m-n)$$

$$= (m^2 - mn) + (3m - 3n)$$
$$= m(m-n) + 3(m-n)$$
$$= (m-n)(m+3)$$

Practice

Factor each expression using grouping:

a) $xy + 2y + 5x + 10$

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b) $3a^2 + 6ab + ab + 2b^2$

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c) $x^3 - x^2y + xy - y^2$

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3. Factoring Trinomials

Key Concept: A trinomial in the form $ax^2 + bx + c$ can often be factored into two binomials.

$(x + *)(x - *)$

A. Factoring Trinomials with Leading Coefficient 1: $x^2 + bx + c$

Process:

1. Find two numbers that multiply to give c and add to give b
2. Use these numbers to write the factored form: $(x + p)(x + q)$ where $p \times q = c$ and $p + q = b$

$b=7 \quad c=12$

Example: Factor $x^2 + 7x + 12$

$(x + \boxed{p})(x + \boxed{q})$

$(x + 3)(x + 4)$

12: 1 x 12
2 x 6
3 x 4

B. Factoring Trinomials with Leading Coefficient $\neq 1$: $ax^2 + bx + c$

Process (AC Method):

1. Multiply $a \times c$
2. Find two numbers that multiply to give $a \times c$ and add to give b
3. Rewrite the middle term using these two numbers
4. Factor by grouping

1) $ac = 3 \cdot 15 = 45$
2) p, q such that
$p \times q = 45$
$p + q = 14$
45: 1 x 45
3 x 15
9 x 5
3)

$3x^2 + 14x + 15 = (x+3)(3x+5)$
$= 3x^2 + 9x + 5x + 15 =$
$= 3x(x+3) + 5(x+3)$
$= (x+3)(3x+5)$

Example: Factor $3x^2 + 14x + 15$
Step 1: Multiply $a \times c =$
Step 2: Find two numbers that multiply to give 45 and add to give 14.

Factors of 45:

Check sums:

Step 3: Rewrite the middle term using 5 and 9.
$3x^2 + 14x + 15 = 3x^2 + 5x + 9x + 15$
Step 4: Factor by grouping.
$3x^2 + 5x + 9x + 15 =$

C. Factoring Perfect Square Trinomials

Perfect Square Trinomials Formulas:

$$\boxed{\begin{aligned} a^2 + 2ab + b^2 &= (a+b)^2 \\ a^2 - 2ab + b^2 &= (a-b)^2 \end{aligned}}$$

~~$(a+b)^2 = a^2 + b^2$~~

$(a+b)^2 = (a+b)(a+b)$
$= a^2 + 2ab + b^2$

Example 1: Factor $4x^2 - 28x + 49$ $= (2x-7)^2$

$= (2x)^2 - 2(2x)7 + 7^2 = (2x-7)^2$
$= a^2 - 2\,a \cdot b + b^2 = (a-b)^2$

$a = 2x$
$b = 7$

Example 2: Factor $81c^4 + 90c^2d + 25d^2$ $= (9c^2 + 5d)^2$

$a^2 + 2ab + b^2 = (a+b)^2$

$25d^2 = \underbrace{(5d)}_{b}{}^2$

$81c^4 = \underbrace{(9c^2)}_{a}{}^2$

$2ab = 2(9c^2)(5d) =$
$= 90c^2d$