Day 5

8.1 Sequences and Series

1. Sequences

In this section we are going to introduce Sequences and Summation Notation. Roughly speaking, a sequence is an infinite list of numbers. The numbers in the sequence are often written as a_1, a_2, a_3,$\cdots$. The dots mean that the list continues forever.

A simple example is the sequence:

$$\begin{array}{ccccc} 5, & 10, & 15, & 20, & 25, \ldots \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ a_1 & a_2 & a_3 & a_4 & a_5 \ldots \end{array}$$

We can describe the pattern of the sequence displayed above by the following formula:

Example 1: Find the first four terms and the 100th term of the sequence defined by each formula.

1. $a_n = 2n - 1$
2. $b_n = n^2 - 1$
3. $c_n = \dfrac{n}{n+1}$.
4. $d_n = \dfrac{(-1)^{n+1}}{n}$

Example 2: Find the nth term of a sequence whose first several terms are given.

(a) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \cdots$

(b) $-2, 4, -8, 16, -32, \cdots$

2. Recursively defined sequences

Some sequences do not have simple defining formulas like those of the preceding example. The nth term of a sequence may depend on some or all of the terms preceding it. A sequence defined in this way is called recursive.

Example 3 - The Fibonacci Sequence:
Find the first 7 terms of the the sequence defined recursively by
$F_1 = 1$, $F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$.

3. Partial sums and Sigma notation

If we add the terms of a sequence we obtain a **series**. So a series is an infinite sum of numbers which my or may not add up to an actual value. In calculus 2 you will learn how to find what an infinite series adds up to for certain types of series.
A **finite series** is a sum of numbers, which can simply be added up to a total. For both infinite series and finite series we use **Sigma notation**.