

Mth 146: Day 2

7.1 Integration by Parts

1 Part 1: New Concepts & Worked Examples

Our next goal is to learn a method to solve integrals of the type $\int f(x)g(x) dx$ where there is no relationship between f and g , meaning one is not the derivative of the other. Examples: $\int x^2 e^x dx$, $\int x \arctan(x) dx$.

This will be accomplished with Integration by Parts (IBP) which is based on the reversal of the product rule.

Integration by Parts Formula: For differentiable functions u, v , we have

$$\int u(x)v'(x) dx = u(x)v(x) - \int u'(x)v(x) dx$$
$$\int u dv = uv - \int v du.$$

Assuming that the relevant functions are defined on the interval $[a, b]$, calculating definite integrals can be done using integration by parts by the following formula:

Integration by Parts for Definite Integrals

$$\int_a^b u(x)v'(x) dx = u(x)v(x) \Big|_a^b - \int_a^b u'(x)v(x) dx.$$

Example 1. $\int xe^x dx$

Example 2. $\int \theta \sec^2(2\theta) d\theta$

Example 3. $\int e^x \sin(x) dx$

How to choose u and dv wisely

Example 4. $\int \ln(x) \, dx$

Example 5. $\int t(5)^t \, dt$

Example 6. $\int_0^1 \arctan(t) \, dt$

Example 7. $\int x \ln(1+x) \, dx$

2 Part 2: Your Turn — Practice

Try the following individually or in small groups. Final answers are given as footnotes at the bottom of the page — only the final answer, not the steps, so use them to check your work rather than replace it.

1. $\int_0^{\pi/2} x^2 \cos(x) \, dx.$ ¹

2. $\int e^x \arctan(e^x) \, dx.$ ²

3. $\int x^2 \tan(x^3) \, dx.$ ³

¹Answer: $\frac{\pi^2}{4} - 2.$

²Answer: $e^x \arctan(e^x) - \frac{1}{2} \ln(1 + e^{2x}) + C.$

³Answer: $-\frac{1}{3} \ln |\cos(x^3)| + C.$

4. $\int_0^2 e^{-2e^x+2x} dx.$ ⁴

5. $\int \frac{\sin(\ln(x))}{x^2} dx.$ ⁵

⁴Answer: $\frac{3}{4e^2} - e^{-2e^2} \left(\frac{e^2}{2} + \frac{1}{4} \right).$

⁵Answer: $-\frac{\sin(\ln x) + \cos(\ln x)}{2x} + C.$

3 Part 3: Reflection

For each learning outcome below, rate how comfortable you feel right now, and write a few sentences: What clicked? What's still fuzzy? What would help?

1. Understand the connection between the product rule and the method of integration by parts.

Comfort level (circle one): 1 — not comfortable 2 3 4 5 — very comfortable

Reflection:

2. Apply integration by parts to both definite and indefinite integrals.

Comfort level (circle one): 1 — not comfortable 2 3 4 5 — very comfortable

Reflection:

3. Calculate integrals which require more than one application of integration by parts.

Comfort level (circle one): 1 — not comfortable 2 3 4 5 — very comfortable

Reflection:
