

In this section, you will study three additional categories of trigonometric identities.

1. The first category involves *functions of multiple angles* such as $\sin(ku)$ and $\cos(ku)$.
2. The second category involves *squares of trigonometric functions* such as $\sin^2 u$.
3. The third category involves *functions of half-angles* such as $\sin\left(\frac{u}{2}\right)$.

1 Double-Angle Formulas

The double-angle formulas allow us to express trigonometric functions of twice an angle in terms of functions of the original angle.

$$\begin{aligned}\sin(2\theta) &= 2 \sin \theta \cos \theta \\ \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta \\ \tan(2\theta) &= \frac{2 \tan \theta}{1 - \tan^2 \theta}\end{aligned}$$

Example 1 Solving a Multiple-Angle Equation

Solve $2 \cos x + \sin(2x) = 0$.

Solution:

Example 2 Evaluating Functions Involving Double Angles

Use the following to find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$.

$$\cos \theta = \frac{5}{13}, \quad \frac{3\pi}{2} < \theta < 2\pi$$

Solution:

2 Power-Reducing Formulas

The power-reducing formulas express powers of sine and cosine in terms of cosines of multiple angles.

$$\begin{aligned}\sin^2 \theta &= \frac{1 - \cos(2\theta)}{2} \\ \cos^2 \theta &= \frac{1 + \cos(2\theta)}{2} \\ \tan^2 \theta &= \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}\end{aligned}$$

Example 4 Reducing a Power

Rewrite $\sin^4 x$ as a sum of first powers of the cosines of multiple angles.

Solution:

3 Half-Angle Formulas

The half-angle formulas express trigonometric functions of half an angle in terms of functions of the original angle.

$$\begin{aligned}\sin\left(\frac{\theta}{2}\right) &= \pm\sqrt{\frac{1 - \cos\theta}{2}} \\ \cos\left(\frac{\theta}{2}\right) &= \pm\sqrt{\frac{1 + \cos\theta}{2}} \\ \tan\left(\frac{\theta}{2}\right) &= \frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta}\end{aligned}$$

The sign in the first two formulas depends on the quadrant in which $\frac{\theta}{2}$ lies.

Example 5 Using a Half-Angle Formula

Find the exact value of $\sin 105^\circ$.

Solution:

Example 6 Solving a Trigonometric Equation

Find all solutions of $1 + \cos^2 x = 2 \cos^2\left(\frac{x}{2}\right)$ in the interval $[0, 2\pi)$.

Solution: