Group 1: Integration by Parts

Your task: As a group, create an overview of this topic to present to the class. Include the key formulas, when to use the technique, a worked example, and common mistakes. Use the hints below as a guide.

Hints: what to include

- Write down the formula:

$$\int u\,dv = uv - \int v\,du, \qquad \int_a^b u\,dv = \Big[uv\Big]_a^b - \int_a^b v\,du.$$

- Explain **LIPET** (Logs, Inverse trig, Polynomials, Exponentials, Trig) as a guide for choosing u. The function that comes first is usually u.
- Show how to choose u and dv, then find du and v. Point out that dv must be something you can integrate.
- Discuss when you must use IBP **more than once**, for example $\int (x^2 + 7x)\cos x\,dx$. Mention that the power of the polynomial tells you how many times.
- Watch out for **cyclic** problems like $\int e^{8\theta}\sin(9\theta)\,d\theta$: after two rounds of IBP the original integral reappears, so move it to the other side and solve for it. Stay consistent with your choice of u in both rounds.
- Mention "hidden" IBP problems where $dv = dx$, such as $\int \ln x\,dx$ or $\int \cos^{-1}(x)\,dx$.
- Mention problems that need a substitution first, such as $\int x\ln(3+x)\,dx$.
- Remember the $+C$, and for definite integrals evaluate uv at the bounds.

Example to present: ______________________________

Group 2: Trigonometric Integrals

Your task: As a group, create an overview of this topic to present to the class. Include the key formulas, when to use the technique, a worked example, and common mistakes. Use the hints below as a guide.

Hints: what to include

- List the key identities:

$$\sin^2 x + \cos^2 x = 1, \qquad \tan^2 x + 1 = \sec^2 x,$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x), \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x), \qquad \sin 2x = 2 \sin x \cos x.$$

- $\int \sin^m x \cos^n x \, dx$**:**
  - If the power of **cosine is odd**: save one $\cos x$, convert the rest to sine, and let $u = \sin x$.
  - If the power of **sine is odd**: save one $\sin x$, convert the rest to cosine, and let $u = \cos x$.
  - If **both powers are even**: use the half-angle identities (possibly more than once).
- $\int \tan^m x \sec^n x \, dx$**:**
  - If the power of **secant is even**: save $\sec^2 x$, convert the rest to tangent, and let $u = \tan x$.
  - If the power of **tangent is odd**: save $\sec x \tan x$, convert the rest to secant, and let $u = \sec x$.
- Know these by heart:

$$\int \tan x \, dx = \ln|\sec x| + C, \qquad \int \sec x \, dx = \ln|\sec x + \tan x| + C,$$

  and how to do $\int \sec^3 x \, dx$ (by IBP).
- Mention product-to-sum formulas for problems like $\int \cos(5t)\cos(10t)\, dt$.
- Watch out: sometimes rewriting in terms of sin and cos first makes the problem easy, for example $\int \tan^6 x \cos^7 x \, dx$.
- Don't forget to convert back to x and include $+C$.

Example to present: ____

Group 3: Trigonometric Substitution

Your task: As a group, create an overview of this topic to present to the class. Include the key formulas, when to use the technique, a worked example, and common mistakes. Use the hints below as a guide.

Hints: what to include

- Make the table of the three cases:

Expression	**Substitution**	**Identity used**
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

- Don't forget to replace dx as well (for example $x = a\tan\theta \Rightarrow dx = a\sec^2\theta\,d\theta$).
- Show how to **draw and label the right triangle** and use it to convert the answer back to x.
- If there is a coefficient in front of x^2 (like $\sqrt{4x^2 - 25}$), adjust: $2x = 5\sec\theta$.
- For expressions like $\sqrt{x^2 + 2x}$ or $\sqrt{3 + 2x - x^2}$, **complete the square** first.
- For **definite integrals**, change the bounds to θ values (then you don't need the triangle).
- Watch out: check whether a simple u-substitution works first! For example,

$$\int \frac{x}{\sqrt{x^2 - 5}}\,dx \quad \text{only needs } u = x^2 - 5.$$

- After substituting, you usually end up with a trig integral (Group 2 skills!).

Example to present: ____

Group 4: Partial Fractions and Rationalizing Substitutions

Your task: As a group, create an overview of this topic to present to the class. Include the key formulas, when to use the technique, a worked example, and common mistakes. Use the hints below as a guide.

Hints: what to include

- **Step 1:** If the degree of the numerator is $\geq$ the degree of the denominator, do **long division** first.
- **Step 2:** Factor the denominator completely.
- **Step 3:** Write the form of the decomposition. Give an example of each case:
  - Distinct linear factors: $\dfrac{A}{x-1} + \dfrac{B}{x+2}$
  - Repeated linear factors: $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1}$
  - Irreducible quadratic factors: $\dfrac{Ax+B}{x^2+25}$
  - Repeated irreducible quadratics: $\dfrac{Ax+B}{x^2+8} + \dfrac{Cx+D}{(x^2+8)^2}$
- **Step 4:** Solve for the constants. Show both methods: plugging in convenient x-values and comparing coefficients.
- **Step 5:** Integrate each piece. Useful formulas:

  $$\int \frac{dx}{x-a} = \ln|x-a| + C, \qquad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$$

  For $\dfrac{Bx+C}{x^2+a^2}$, split it into two integrals (one u-sub and one $\tan^{-1}$).
- **Rationalizing substitutions:** if the integrand has $\sqrt[n]{g(x)}$, let $u = \sqrt[n]{g(x)}$. Solve for x and find dx in terms of u. For example, $u = \sqrt{x-1}$ gives $x = u^2+1$ and $dx = 2u\,du$. With $\sqrt{x}$ and $\sqrt[3]{x}$ together, use $u = \sqrt[6]{x}$.
- Also mention substitutions like $u = e^x$ or $u = \tan t$ that turn the integral into a rational function.
- Watch out: remember absolute values in the logs, and substitute back to x at the end.

Example to present: ____________________

Group 5: Improper Integrals of Type I (including p-integrals)

Your task: As a group, create an overview of this topic to present to the class. Include the key formulas, when to use the technique, a worked example, and common mistakes. Use the hints below as a guide.

Hints: what to include

- Define a Type I improper integral: the **interval is infinite**. Contrast this with Type II, where the function has an infinite discontinuity.
- Write the definitions using limits:

$$\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx, \qquad \int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx.$$

- For $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx$, split at a point (for example 0). Both pieces must converge; if either one diverges, the whole integral diverges.
- Define **convergent** (the limit exists and is finite) and **divergent** (the limit is $\pm\infty$ or does not exist, e.g. it oscillates like $\cos t$).
- State the **p-integral** result:

$$\int_1^\infty \frac{1}{x^p}\,dx \ \text{ is } \begin{cases} \text{convergent, equal to } \dfrac{1}{p-1}, & p>1,\\ \text{divergent,} & p\le 1.\end{cases}$$

- Useful limits to know: $\displaystyle\lim_{t\to\infty} e^{-t}=0,\ \lim_{t\to\infty}\ln t=\infty,\ \lim_{t\to\infty}\tan^{-1} t=\frac{\pi}{2}$.
- Use **L'Hôpital's rule** for limits like $\displaystyle\lim_{t\to\infty}\frac{\ln t}{t}$ or $\displaystyle\lim_{t\to\infty} te^{-t}$, which show up after integration by parts.
- Watch out: always write the limit notation. Never plug ∞ in directly.
- Show one example that needs another technique first, such as IBP ($\int_1^\infty \frac{\ln x}{x^2}\,dx$) or partial fractions ($\int_3^\infty \frac{dx}{x^2+x}$).

Example to present: ______________________________