Math 142: Monday, Nov 11th, Day 23 Notes

Agenda for Monday, Nov 11th 2024

Coming up

- Student Hours Tuesday 1:215pm in C162
- HW 16 (on confidence intervals) will be posted today
- Excel Assignment 4 due Wednesday
- Deadline to withdraw from class is Wednesday
- Exam 3 on Wed Nov 20th

Recall: Confidence interval ~~for~~ for mean when σ is known

Formula for a confidence interval for a population mean (μ) when σ is known, is:

$$\text{CI} = \bar{x} \pm \underbrace{z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}_{\text{margin of error (ME)}}$$

where

$\bar{x}$ - sample mean (a point estimate)

$$\boxed{\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\ ,\ \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)}$$

σ - population st. dev;

n - sample size;

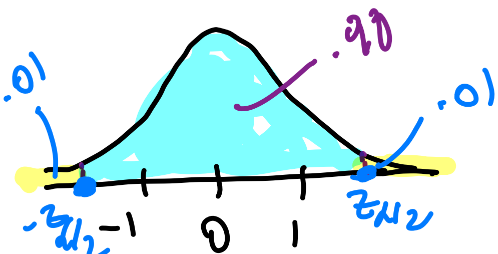
$z_{\alpha/2}$ - critical value depending on the level of confidence we want

Example 1. Find $z_{\alpha/2}$ for a 98% confidence interval.

$z_{\alpha/2}$ for 98% CI = ? = 2.33

$-z_{\alpha/2}$ = invNorm(.01, 0, 1) = −2.33

$N(\mu=0, \sigma=1)$



If $\mu=0$ & st. dev $\sigma=1$ this is called a standard normal curve.

Practice:

Example 2. Find the 98% confidence interval for the population mean, μ. Assume the distribution is normally distributed with a population standard deviation of 6 minutes. A random sample of 28 pizza delivery restaurants is taken and has a sample mean delivery time of 36 minutes.

STAT → TEST ↓ ZInterval

$\bar{x} = 36$ $n = 28$

$\sigma = 6$ CI 98% $z_{\alpha/2} = 2.33$

CI: (33.362, 38.638)

We are 98% confident that the true pop. mean μ is between 33.36 minutes and 38.64 minutes.

Calculator instructions to find confidence interval

Have this information ready first:

- Sample mean ($\bar{x}$) ✓
- Population standard deviation (σ) ✓
- Sample size (n) ✓
- Confidence level (typically 90%, 95%, or 99%)

Then press **STAT**, use arrow right to go to **TESTS** and then go down until you find **ZInterval**.

Example 3. Use your calculator to find the 90% confidence interval for the population mean, μ. Assume the distribution is normally distributed with a population standard deviation of 6 minutes. A random sample of 10 pizza delivery restaurants is taken and has the following raw data in minutes:
30, 28, 41, 34, 29, 32, 36, 39, 34, 37

$\boxed{L_1}$ STATS → TESTS → ZInterval → DATA
↳ L1

Finding Sample Sizes n, to achieve a desired margin of error

Sometimes we are interested in achieving a certain margin of error, ME and we want to know how big our sample size, n, should be to achieve that.
Formula for margin of error:

$$ME = z_{\alpha/2}\frac{\sigma}{\cancel{\sqrt{n}}} \quad | \cdot \cancel{\sqrt{n}}$$

$$\sqrt{n}\, ME = z_{\alpha/2}\,\sigma \quad | \div ME$$

$$\sqrt{n} = \frac{z_{\alpha/2}\,\sigma}{ME} \quad | \text{ square both sides}$$

$$(\cancel{\sqrt{n}})^{\cancel{2}} = \left(\frac{z_{\alpha/2}\sigma}{ME}\right)^2$$

$$\boxed{n = \left(\frac{Z_{\alpha/2}\cdot\sigma}{ME}\right)^2}$$

Example 4. A cheese processing company wants to estimate the mean cholesterol content of all one-ounce servings of cheese. The estimate must be **within** 0.5 milligram of the population mean. Determine the minimum required sample size to construct a 95% confidence interval for the population mean. Assume the population standard deviation is 2.8 milligrams.

$n = ?$

$\sigma = 2.8$

$z_{\alpha/2} = 1.96$ (for 95% CI)

$ME = 0.5$

Alway round up to nearest integer

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{ME}\right)^2 = \left(\frac{(1.96)(2.8)}{0.5}\right)^2$$

$n = 121$

$n = 80$

Confidence intervals for population means when the standard deviation of the population is not known

In practice, we rarely know the population standard deviation, σ. We can use the sample standard deviation s as an estimate for σ and proceeded as before to calculate a confidence interval with close enough results. Up until the mid-1970s, some statisticians used the normal distribution approximation for large sample sizes and used the **Student's t-distribution** only for sample sizes of at most 30. With graphing calculators and computers, the practice now is to use the **Student's t- distribution** whenever s is used as an estimate for σ.

t-distribution vs normal distribution:

Confidence interval formula when σ is unknown

$$\text{CI} = \bar{x} \pm \underbrace{t^* \times \frac{s}{\sqrt{n}}}_{\text{ME}}$$

s - sample st.dev

estimates μ to be in $\left(\bar{x} - t^*\frac{s}{\sqrt{n}}, \bar{x} + t^*\frac{s}{\sqrt{n}}\right)$

$\boxed{t^*}$ corresponds to a certain confidence level and on degree of freedom $\boxed{df = n-1}$.

Then press **STAT**, use arrow right to go to **TESTS** and then go down until you find **TInterval**.

Example

Find the 95% confidence interval for a population mean given the following information:

- Sample mean is $\bar{x} = 75$ and sample standard deviation is $s = 8$
- Sample size: $n = 30$

$\boxed{(72.013, 77.987)}$

Lets see by hand: $\left(75 - t^* \frac{8}{\sqrt{30}}, 75 + t^* \frac{8}{\sqrt{30}}\right)$

$t^* = 2.045$

Confidence interval for proportions

$\left(75 - 2.045 \frac{8}{\sqrt{30}}, 75\right.$

$(72.01, 77.92)$

Example 5. For a class project, a political science student at a large university wants to estimate the percent of students who are registered voters. The student surveys 500 students and finds that 59.7% are registered voters. Compute a 90% confidence interval for the true percent of students who are registered voters, and interpret the confidence interval.