MTH 144 Last Group Quiz : Verifying Identities

Student Names (Circle your own name): _____

Verify the following trigonometric identities algebraically.

use: $1+\cot^2 x = \csc^2 x$ P3 $\cot^2 x = \csc^2 x - 1$

1. $\tan^2 y(\csc^2 y - 1) = 1$

$\tan^2 y \cdot \cot^2 y = \tan^2 y \cdot \frac{1}{\tan^2 y} = 1$

2. $\frac{\cot\alpha}{\csc\alpha - 1} = \frac{\csc\alpha + 1}{\cot\alpha}$ (do not assume true and cross multiply)

$\frac{\cot\alpha}{\csc\alpha - 1} = \frac{\cot\alpha}{\csc\alpha - 1}\frac{\csc\alpha + 1}{\csc\alpha + 1} = \frac{\cot\alpha(\csc\alpha+1)}{\csc^2\alpha - 1} = \frac{\cot\alpha(\csc\alpha+1)}{\cot^2\alpha}$

$= \frac{\csc\alpha + 1}{\cot\alpha}$

3. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$ common denominator

$\frac{1+\sin\theta}{\cos\theta}\frac{(1+\sin\theta)}{(1+\sin\theta)} + \frac{\cos\theta}{1+\sin\theta}\frac{\cos\theta}{\cos\theta} = \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{1 + 2\sin\theta + \overbrace{\sin^2\theta + \cos^2\theta}^{=1}}{\cos\theta(1+\sin\theta)} = \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$= \frac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}} = \frac{2}{\cos\theta} = 2\sec\theta$

4. $\sin\theta \csc\theta - \sin^2\theta = \cos^2\theta$

$\sin\theta \csc\theta - \sin^2\theta = \sin\theta \frac{1}{\sin\theta} - \sin^2\theta = \frac{\sin\theta}{\sin\theta} 1 - \sin^2\theta$

$= 1 - \sin^2\theta = \cos^2\theta$

5. $\sin^2\left(\frac{\pi}{2} - x\right) + \sin^2 x = 1$

use cofunction $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

so $\sin^2\left(\frac{\pi}{2} - x\right) = \cos^2 x$

$\cos^2 x + \sin^2 x = 1$

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① $\sqrt{a^2 - u^2} = \sqrt{a^2 - (a\sin\theta)^2} = \sqrt{a^2 - a^2\sin^2\theta}$

$= \sqrt{a^2(1 - \sin^2\theta)}$

$= \sqrt{a^2}\sqrt{1 - \sin^2\theta}$

$= a\cos\theta$

② $\sqrt{a^2 + u^2} = \sqrt{a^2 + a^2\tan^2\theta} = \sqrt{a^2(A\tan^2\theta)}$

$= a\sec\theta$

4. $\sin\theta \csc\theta - \sin^2\theta = \cos^2\theta$

5. $\sin^2\left(\frac{\pi}{2} - x\right) + \sin^2 x = 1$

6. $\frac{\cos\theta}{1 - \sin\theta} = \sec\theta + \tan\theta$