Day 8 Notes
Chapter 3 Functions and Relations

1. Functions and Relations

In nearly every physical phenomenon, we observe that one quantity depends on another. For example, the cost of tuition depends on the number of credits you take, or the area of a circle depends on its radius.

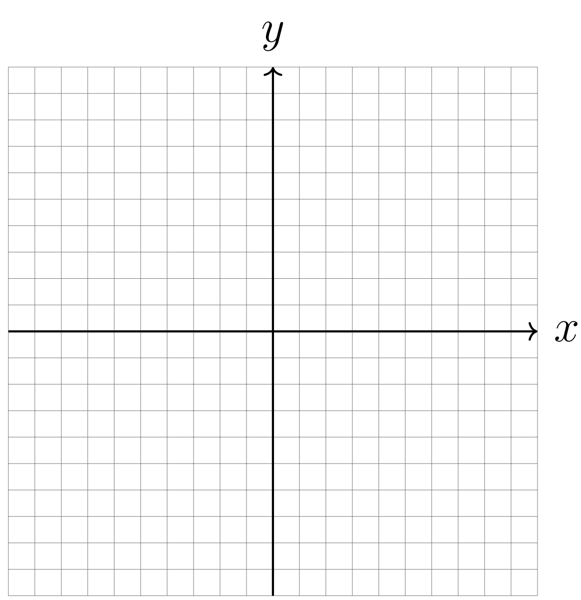
We use the term **function** to describe this dependence of one quantity on another.
We will use letters such as $f, g, h \cdots$ to represent functions.
We can represent a function in **four different ways**. For example, we can use the letter f to represent a rule that doubles the input and then add 1.

1. **Verbally:** f is the rule that doubles the input x and then adds 1.

2. **Algebraically**:

3. **Numerically:**

4. **Graphically**:



Formal definition of a function:
A function f is a rule that assigns to ______ element x in a set A
____________ element, called $f(x)$, in a set B.
Set A is called the ________________ of the function and it consists of

____

Set B is called the________________ of the function and it consists
________________, as x varies throughout the domain.

Example 1. (Interval notation for Domains and Range) Find the domain and range for $f(x) = \sqrt{x+2}$

Example 2. Identify if the following relations are functions. If yes, identify the domain and range.

1. $y = x^2 + 1$

2.

x	y
1	2
1	3
2	4

3.

x	y
1	5
2	7
3	5

2. Piecewise defined functions

Sometimes a function has different rules for different portions of the domain and when that happens we call those **piecewise defined functions**.

Example 3. So, for example, if a cell phone company charges $40 a month if you use less than 500 minutes, but for every minute beyond the allowed 500 minutes you get charged a quarter a minute, the function that calculates the cost of the phone plan will be a piecewise defined function.
Find a formula for the cost of this phone plan, $C(x)$ where x is the number of minutes used in a month period.

Example 4. Recall the absolute value function $f(x) = |x|$.

1. Graph f.

2. Rewrite f as a piecewise defined function.

3. Evaluating a Function

Evaluating a function at a specific input value means plug in that specific value in the formula to get your output.

Example 5. Let $f(x) = 2x^2 + 3x + 1$. Evaluate

1. $f(2)$

2. $f(-3)$

4.Difference Quotient

The difference quotient represents the average rate of change of a function between two values x and $x + h$.

Recall that the Average rate of change of a function f between two points x_1 and x_2 is just the slope of the line going through the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$.

Formula:

Example 6. Find the difference quotient for $f(x) = 2x^2 + 3x + 1$.

Example 7. Find the difference quotient for $f(x) = \dfrac{1}{x+1}$.

Practice 1: Find the domain of functions:

1. $f(x) = \dfrac{\sqrt{2+x}}{3-x}$

2. $g(x) = \sqrt[3]{x-1}$

3. $h(x) = \dfrac{x^4}{(x-3)(x+3)}$

4. $f(t) = \sqrt{t^2 - 2t - 8}$

Practice 2: Evaluate (if possible) the function $g(x) = \dfrac{1-x}{1+x}$ at:

1. $g(2)$
2. $g(-1)$
3. $g(a-1)$
4. $g(x^2-1)$

Practice 3: Find the difference quotient for $f(x) = x^2 + 2x - 3$.

Practice 4: Evaluate the piecewise defined function at the indicated values:

$$f(x) = \begin{cases} x^2 + 2x & x \leq -1 \\ x & -1 < x \leq 1 \\ -1 & 1 < x \end{cases}$$

1. $f(-4) =$
2. $f(25) =$
3. $f(-1) =$
4. $f(0) =$