Day 7 notes

8.3 Geometric Sequences

1. Geometric Sequences

Next, we are going to introduce Geometric Sequences. A geometric sequence is generated when we start with a number a and repeatedly multiply by a fixed nonzero constant r.

DEFINITION OF A GEOMETRIC SEQUENCE

A **geometric sequence** is a sequence of the form

$$a, ar, ar^2, ar^3, ar^4, \ldots$$

The number a is the **first term**, and r is the **common ratio** of the sequence. The ***n*th term** of a geometric sequence is given by

$$a_n = ar^{n-1}$$

Example 4: If $a = 3$ and $r = 2$, then we have the geometric sequence:

Example 6: The sequence
$2, -10, 50, -250, 1250, \cdots$ is a geometric sequence with $a = 2$ and $r = -5$. Find a_{10}.

Example 6: Find the common ratio, the first term, the nth term, and the eighth term of the geometric sequence $5, 15, 45, 135, \cdots$

2. Partial Sums of Geometric Sequences

The partial sum s_n of a geometric sequence is given by the formula below.

PARTIAL SUMS OF A GEOMETRIC SEQUENCE

For the geometric sequence defined by $a_n = ar^{n-1}$, the ***n*th partial sum**

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} \qquad r \neq 1$$

is given by

$$S_n = a\frac{1 - r^n}{1 - r}$$

Example 7: Find the following partial sum of a geometric sequence:
$1 + 4 + 16 + \cdots + 4096$

3. Infinite Geometric Series

An infinite geometric series is a series of the form
$a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^{n-1} + \cdots$.

SUM OF AN INFINITE GEOMETRIC SERIES

If $|r| < 1$, then the infinite geometric series

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \cdots$$

converges and has the sum

$$S = \frac{a}{1 - r}$$

If $|r| \geq 1$, the series diverges.

Example 8: Determine whether the infinite geometric series is convergent or divergent. If it is convergent, find its sum.

(a) $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \cdots$

(b) $1 + \frac{7}{5} + \frac{49}{25} + \cdots$