Example 4. Classical probability Experiment: Rolling a fair 12 sided die.

S =

Event A: rolling a prime number A =

Event B: rolling a number divisible by 5 B =

Event C: rolling a number greater than 4 which is divisible by 3

C =

Find the following probabilities.

P(A) =

P(B) =

P(C) =

previous class

Day 12 notes:

Empirical (Experimental) Probability used when events are not equally likely (or it's the long term relative frequency).

$$P(A) = \frac{\text{number of time A happens}}{\text{number of total trials}} = \frac{\text{frequency}}{\text{sample size}}$$

Example 5. Julia made a die for a game. It had 5 faces, labeled 1-5. Because of the shape of the die, it was biased in favor of rolling a 3. She experimented to estimate how likely it was for a 3 to be rolled. Julia rolled the die 100 times, and the number of times she rolled a 3 was 45.

$$P(3) = \frac{45}{100} = 0.45$$

Example 6. Empirical Probability: Choosing a student from a survey

	Accounting	Non Accounting	
Freshman	20	50	70
Soph	14	56	70
	34	106	140

$S = \{FA, FN, SA, SN\}$

sample size = 140

1. What is the probability of choosing a student who is majoring in accounting? A

$P(A) = \frac{34}{140} = \frac{\text{frequency}}{\text{sample size}} = 0.2428 = 0.243$

2. What is the probability of choosing a student who is a sophmore?

$P(\text{Soph}) = \frac{70}{140} = \frac{1}{2}$

$P(FAcc) = \frac{20}{140} = 0.143$

3. What is the probability of choosing a student who is not majoring in accounting?

$P(\text{not Acc}) = \frac{106}{140} = 0.757$

$P(\text{not Acc}) = \frac{140 - 34}{140} = \frac{140}{140} - \frac{34}{140} = 1 - P(\text{Acc})$

Properties of Probability

1. For any event A, $\mathbf{0 \le P(A) \le 1}$. That is, the probability of any event is always a number between 0 and 1.
2. The probability of any event A is equal to the sum of the probabilities of the individual outcomes in A.
3. The sum of the probabilities of all outcomes in S must equal 1. This is true whether S consists of equally likely outcomes.

(2) Rolling a 6 sided die

$S = \{1, 2, 3, 4, 5, 6\}$

A = even number

$A = \{2, 4, 6\}$

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(2) + P(4) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$

(3) $P(\text{sample space}) = 1$

Math 140: Thursday, Oct 3, Day 12 Notes

Agenda for Thursday, Oct 3,2024

Coming up

- ! First Probability Homework will be posted this evening
- Office Hour **today 1-2:15pm** in C162

$P(\text{K and } \heartsuit) = \frac{1}{52}$

Consider a standard deck of cards.

face cards

2♡	3♡	4♡	5♡	6♡	7♡	8♡	9♡	10♡	J♡	Q♡	K♡	A♡	13
2♢	3♢	4♢	5♢	6♢	7♢	8♢	9♢	10♢	J♢	Q♢	K♢	A♢	13
2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣	A♣	13
2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠	A♠	13

52

high cards

$P(\heartsuit) = \frac{13}{52} = \frac{1}{4}$; $P(\text{face card}) = \frac{12}{52}$

Probability Basic Concepts

Often we deal with events that are described in terms of other events. These are called compound events and they involve the connectors *or*, *and* and *not*.

not : Complement Rule

$P(A)$ → prob. of event A

$P(\text{not } A) = P(A') = 1 - P(A)$

A' prime & it means not event A

Probabilities involving OR

Example 1. Consider the experiment of rolling a six sided die and flipping a coin.

Sample space: $S = \{1T, 1H, 2T, 2H, 3T, 3H, 4T, 4H, 5T, 5H, 6T, 6H\}$.

Let event A be the event of rolling an even number and event B be the event of getting Tail on the coin.

$A = \{ 2T, 2H, 4T, 4H, 6T, 6H \}$ $n(A) = 6$

$B = \{ 1T, 2T, 3T, 4T, 5T, 6T \}$ $n(B) = 6$

The event A OR B includes $n(A \text{ or } B) = 9$

event A or B = $\{ 2T, 2H, 4T, 4H, 6T, 6H, 1T, 3T, 5T \}$

↓ combine both events, but don't write duplicates

even A and B = $\{ 2T, 4T, 6T \}$

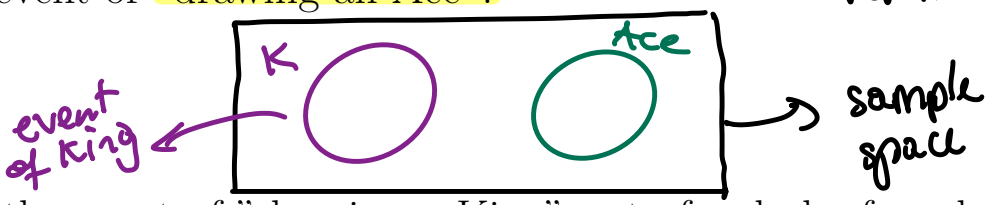
Mutually exclusive events are events that have nothing in common (or they can't happen at the same time).

Example 2. Are the following events mutually exclusive?

1. Let A be the event of "drawing a King" out of a deck of cards and B be the event of "drawing an Ace".

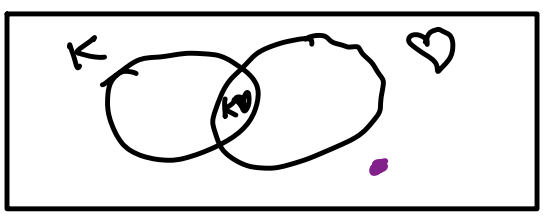
   Venn diagram

   1 yes

2. Let A be the event of "drawing a King" out of a deck of cards and B be the event of "drawing a Heart".

   NO

3. Let A be the event of "getting an A" and B the event of "getting a B" in this class.

yes

Addition Rule for mutually exclusive events:

For two mutually exclusive events A and B, the probability that one **or** the other occurs is the sum of the probabilities of the two events.

$$P(A \text{ or } B) = P(A) + P(B)$$

The general additional rule

The Addition Rule doesn't work for events that aren't mutually exclusive. If the probability of owning a smartphone is 0.50 and the probability of owning a computer is 0.90, the probability of owning a smartphone **or** a computer may be pretty high, but it is not 1.40!

We'd like to generalize the Addition Rule to include events that aren't mutually exclusive.

General Addition Rule:

For any events A and B, the probability that A **or** B occurs is the sum of the probabilities of the two events minus the probability that both A and B occur.

$$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$9 = 6 + 6 - 3$$

Example 3. A survey of college students asked the questions: "Are you currently in a relationship?" and "Are you involved sports?" The survey found that 33% were currently in a relationship, and 25% were involved in sports. Eleven percent responded "yes" to both.

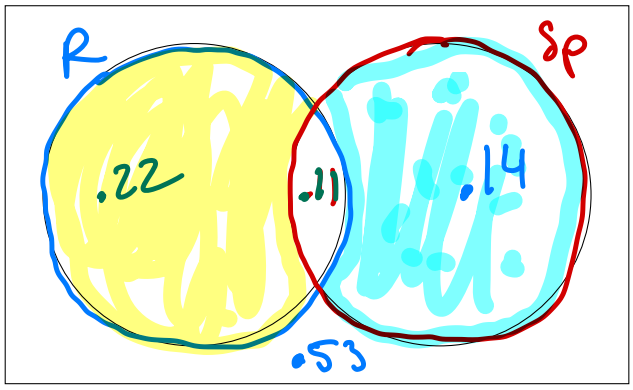
R Sp

1. We can use a Venn Diagram to answer the following questions.

$P(R) = .33$

$P(Sp) = .25$

$P(R \text{ and } Sp) = .11$



$.33 - .11$

$.25 - .11 = .14$

$1 - (.22 + .11 + .14)$

What is the probability that a randomly selected student:

(a) is in a relationship **or** is involved in sports?

(b) is not in a relationship and is not involved in sports?

a) $P(R \text{ or } Sp) = P(R) + P(Sp) - P(R \text{ and } Sp) = .33 + .25 - .11 = .47$

b) $1 - (.22 + .11 + .14) = .53$ (outside of circle region)

(c) is in a relationship but not involved in sports?

.22 yellow region

(d) either is in a relationship or is involved in sports, but not both?

$.22 + .14 = .36$

only R only Sp