Math 142: Monday, Nov 4th, Day 21 Notes

Agenda for Monday, Nov 4th 2024

Coming up

- Student Hours Tuesday 1-2:15pm in C162
- HW 14 due today and HW 15 due Wednesday
- Excel Assignment 4 will be posted soon
- Spring 2025 Priority Registration opens today

Recall:

population : all CLC students

X - height

$\mu = 67$

$\sigma = 3$ inches

$n = 20$

sample

$\bar{x}_1 = 65$

s_1

s = sample st. dev

$\bar{x}_2 = 73$

s_2

Chapter 8 Central Limit Theorem and Sampling Distributions

Central Limit Theorem states 3 things:
Suppose X is a random variable with mean μ and standard deviation σ. Suppose we select random samples of size n. Then the following are true:

- Mean of sample means

  $$\mu_{\bar{x}} = \mu$$

- The standard deviation of the sampling distribution also called **Standard error of mean** is

  $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- Distributions

  1. When original distribution is j-shaped, uniform, non-normal: → skewed, bimodal (dragons)

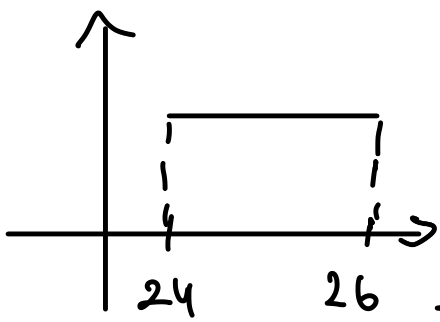
     $\bar{x}$ is approximately normally distributed.

  2. When original distribution is normal: → heights student, weights of bunnies

     $\bar{x}$ are also normally distributed.

Example 1. A manufacturer produces 25 pound dumbbell. The lowest actual weight is 24 pounds and the highest is 26 pounds. Each weight is equally likely so the distributions of weights is **uniform**. A sample of 100 weights is selected.

1. What is the distribution for the weights of one 25-pound lifting weight? What is the mean and standard deviation of this distribution? Draw the distribution. μ



X weight of 1 dumbell

$$\mu = \frac{a+b}{2} = \frac{24+26}{2} = 25 \text{ lbs}$$

$$\sigma = \frac{b-a}{\sqrt{12}} = \frac{2}{\sqrt{12}} = 0.58$$

$$X \sim U(24, 26)$$

2. What is the distribution for the mean weight of one hundred 25-pound lifting weights? What is the mean and standard deviation of the sampling distribution. Draw the distribution. $n = 100$

$$\mu_{\bar{x}} = 25$$

$$\sigma_{\bar{x}} = 0.058 = \frac{\sigma}{\sqrt{n}} = \frac{0.58}{\sqrt{100}} = \frac{0.58}{10}$$

$$\bar{X} \sim N(\mu_{\bar{x}}, \sigma_{\bar{x}})$$

$$N(25, 0.058)$$

Example 2. The amounts of time employees at a large corporation work each day are normally distributed, with a mean of 7.6 hours and a standard deviation of .35 hour. Random samples of size 16 are drawn from the population and the mean of each sample is determined

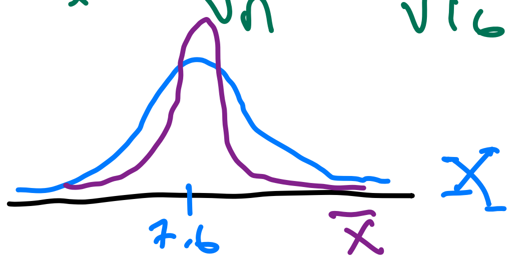
a) Find the mean of the distribution of sample means.

b) Find the standard deviation of the distribution of sample means.

c) Draw the distribution of sample means

a) $\mu_{\bar{x}} = ?$ by CLT $\mu_{\bar{x}} = \mu = 7.6$ hours

b) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.35}{\sqrt{16}} = \frac{0.35}{4} = 0.0875$



Practice in Groups (or on your own at home if we have no time left in class)

try

Example 3. A study found that the mean migration distance of the green turtle was 2200 km and the standard deviation was 625 km.

1. Assuming the distances are normally distributed find the probability that a randomly selected green turtle migrates a distance of less than 1900 km

2. A sample of 12 green turtle is randomly selected. Find the probability that the sample mean of the distance migrated is less than 1900 km.