Day 10,

- Written HW 2 will be posted soon

Day 10 Notes
Thursday Feb 27th
2.3 Functions and Relations

1. Functions and Relations

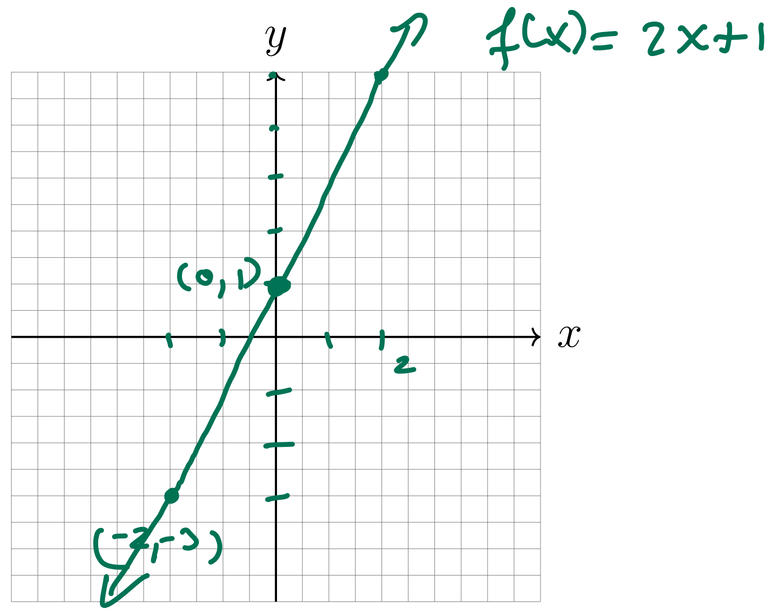
In nearly every physical phenomenon, we observe that one quantity depends on another. For example, the cost of tuition depends on the number of credits you take, or the area of a circle depends on its radius.

We use the term **function** to describe this dependence of one quantity on another.
We will use letters such as $f, g, h \cdots$ to represent functions.
We can represent a function in **four different ways**. For example, we can use the letter f to represent a rule that doubles the input and then add 1.

1. **Verbally:** f is the rule that doubles the input x and then adds 1.
2. **Algebraically**: $y = 2x + 1$ function notation $y = f(x)$
   Let x be the input (independent variable)
   Let y be the output (dependend variable) f of x
3. **Numerically:** $f(x) = 2x + 1$ $\quad 2 \cdot \frac{1}{2} + 1 = 2$
   ↳ make a table with values for x and their corresponding y values.

x	-2	0	$\frac{1}{2}$	3	4
$y = f(x)$	-3	1	2	7	9

4. **Graphically**:
   plot points and connect them

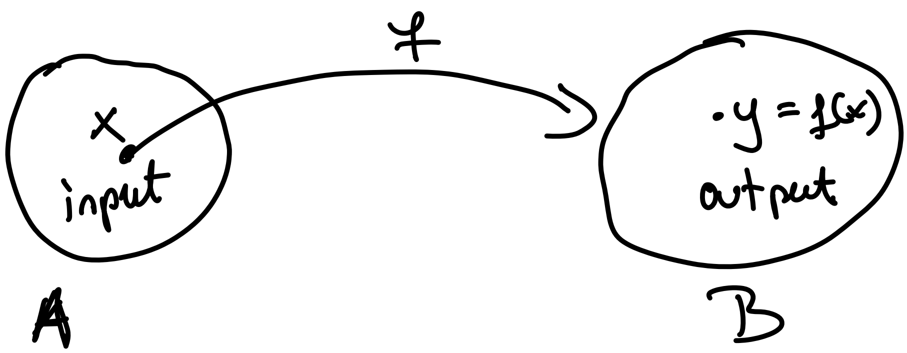


Formal definition of a function:

A function f is a rule that assigns to each element x in a set A exactly one element, called y or $f(x)$, in a set B.

Set A is called the domain of the function and it consists of all valid inputs.

Set B is called the range of the function and it consists all outputs, as x varies throughout the domain.



Example 1. Identify if the following relations are functions. If yes, identify the domain and range.

1. $y = x^2 + 1$ Check that it satisfies the definition of a function.
→ yes a function (each input has exactly one output).
Domain: all real numbers : $(-\infty, \infty)$
Range: $[1, \infty)$

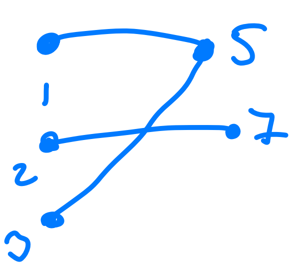
2.

x	y
1	2
1	3
2	4

NO: 1 has two outputs

3.

x	y
1	5
2	7
3	5



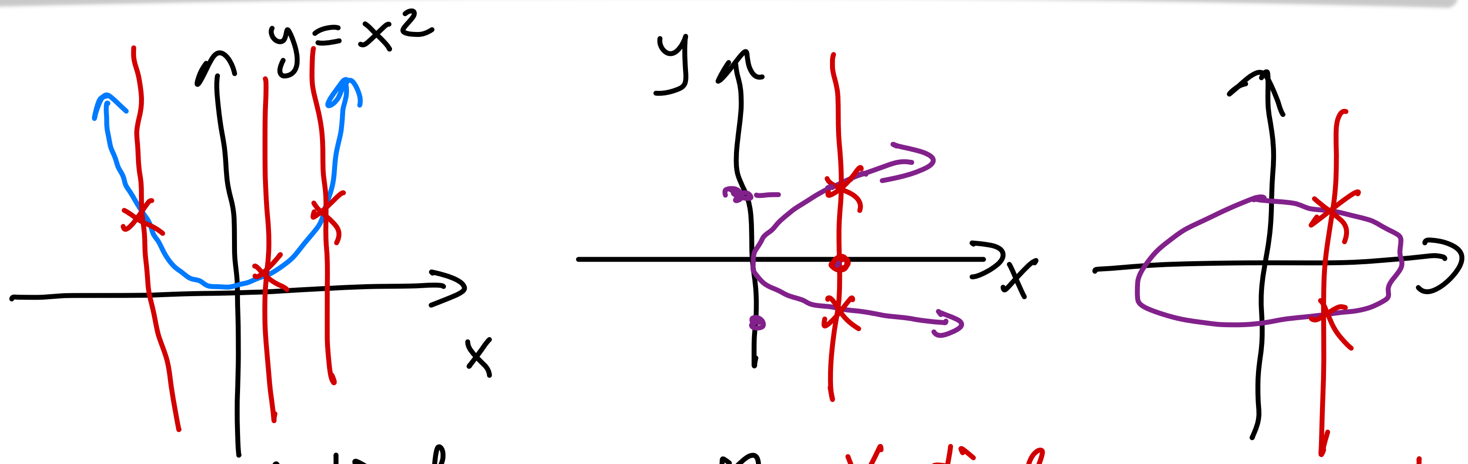
yes

4. $R = \{(3, 1), (4, 2), (5, 3), (3, 7), (6, 4)\}$ NO

5. $S = \{(1, 5), (2, 5), (3, 6), (4, 7), (5, 7)\}$ yes

2. Using the Vertical Line Test

The graphs of three relations are shown below. Determine if each relation defines y as a function of x.



Yes. Vertical line intersect the graph in only one spot.

NO. Vertical line intersects in two spots

NO !

3. Evaluating a Function

Evaluating a function at a specific input value means plug in that specific value in the formula to get your output.

Example 2. Let $f(x) = 2x^2 + 3x + 1$. Evaluate

1. $f(2) = 2(2)^2 + 3(2) + 1 = 8 + 6 + 1 = 15$

2. $f(-3) = 2(-3)^2 + 3(-3) + 1 = 2(9) - 9 + 1 = 10$

3. $f(a+1) = 2(a+1)^2 + 3(a+1) + 1$

$= 2(a+1)(a+1) + 3a + 3 + 1$

$= 2(a^2 + a + a + 1) + 3a + 4$

$= 2a^2 + 4a + 2 + 3a + 4$

$= 2a^2 + 7a + 6$

4.Finding the x anf y intercepts of a function

Calculator:
y: 2 Trace: CALC x=0 f(0)
x: 2 Trace: CALC zero

Given a function define by $y = f(x)$.

- The ~~x~~ zeros intercepts are when it intersects the x axis : to find then you set $y=0$ or $f(x)=0$.
- The y-intercepts are where it intersects the y axis : to find it set $x=0$ or find $f(0)$.

Example 3. Find the intercepts for $f(x) = x^2 - 4$ and $g(x) = |x| - 5$.

f : x-intercepts: set $y=0$ or $f(x)$

$f(x)=x^2-4$

$x^2-4=0$

$x^2=4$

$x=\pm 2$

$(2,0)$ & $(-2,0)$

y-intercept $f(0)=0^2-4=-4$

$(0,-4)$

g

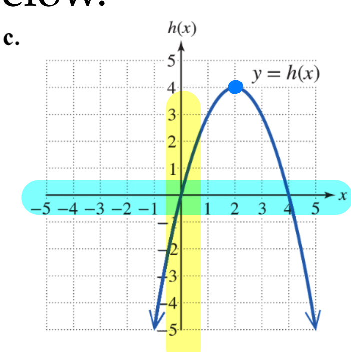
5.Determining Domain and Range

We can determine the domain and range of a function either from the graph or from the formula for the function.

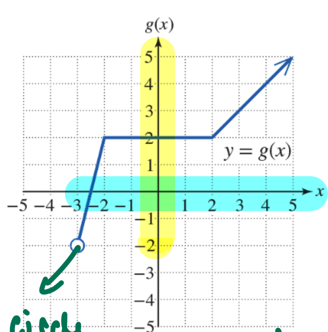
Graphically:

Example 4. Determine the domain and range of the functions graphed below.

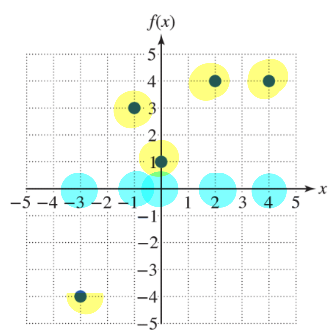
c.



b.



a.



circle means the value is not included

Domain: $(-\infty,\infty)$

Range: $-\infty, 4]$

Domain: $(-3,\infty)$

Range: $(-2,\infty)$

Domain: $\{-3,-1,0,2,4\}$

Range: $\{-4,3,1,4,4\}$

6.Guidelines to finding the Domain of a function algebraically

Given a function defined by $y = f(x)$ to find the implied domain.

- start with all real numbers
- exclude x values that make a denominator
- exclude x values that make the quantity use $\sqrt{\ }$, $\sqrt[4]{\ }$, $\sqrt[6]{\ }$, negative

Algebraically: Find the domain of functions:

1. $f(x) = \dfrac{\sqrt{2+x}}{3-x}$

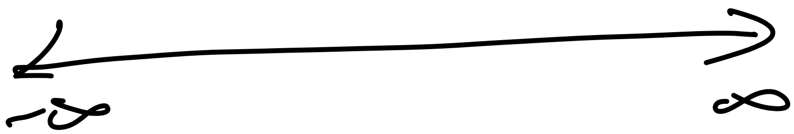
$2+x < 0$

$x < -2$

$\sqrt{2+(-2)} = \sqrt{0} = 0$ ✓

Domain : $[-2,3) \cup (3,\infty)$

2. $g(x) = \sqrt[3]{x-1}$



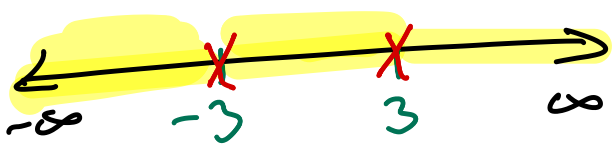
Domain : $(-\infty,\infty)$

$\sqrt[3]{-8} = -2$ bc. $(-2)^3 = (-2)(-2)(-2)$

$= 4(-2) = -8$

3. $h(x) = \dfrac{x^4}{(x-3)(x+3)}$

$x=3$ $x=3$



Domain: $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$

4. $f(t) = \sqrt{t^2 - 2t - 8}$

$= \sqrt{(t-4)(t+2)}$

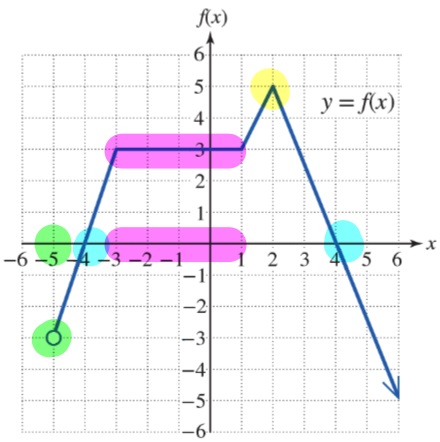
Domain: $(-\infty,-2) \cup (4,\infty)$

Range: $[0,\infty)$

7.Interpret a function Graphically

We can get a lot of information from the graph of a function.

Example 5. Use the function f graphed below to answer the questions.



a. Determine $f(2)$. $= 5$

b. Determine $f(-5)$. = Does not exist

c. Find all x for which $f(x) = 0$. (where it crosses x axis: $-4, 4$

d. Find all x for which $f(x) = 3$. for all x in $[3,1]$

e. Determine the x-intercept(s). $(-4,0)$ and $(4,0)$

f. Determine the y-intercept. $(0,3)$

g. Determine the domain of f. $(-5, \infty)$

h. Determine the range of f. $(-\infty, 5]$