

Day 9 Notes

Graphs of Functions

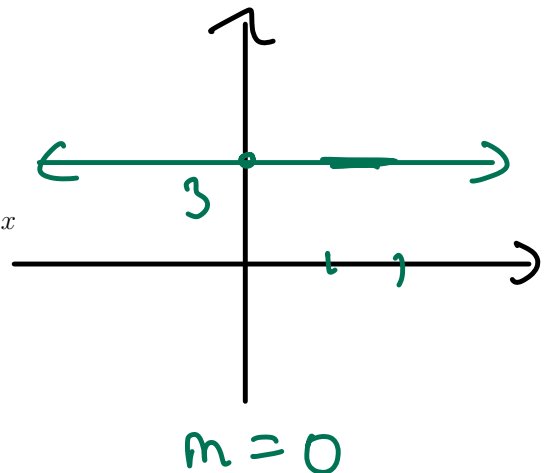
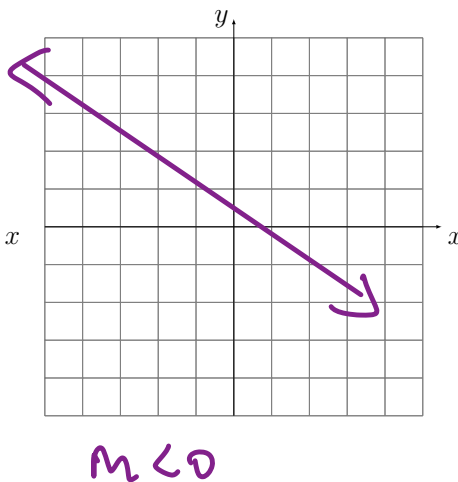
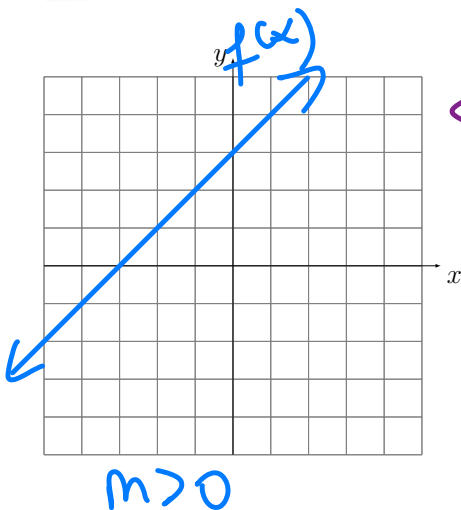
1. Graphs of Functions: Introduction

The most important way to visualize a function is through its graph. In this section we investigate in more detail the concept of graphing functions.

To graph of a function f , we plot the points (x, y) in a coordinate plane where the x coordinate represents an input and the y coordinate is the corresponding output of the function, $y = f(x)$.

Examples of some functions and their graphs.

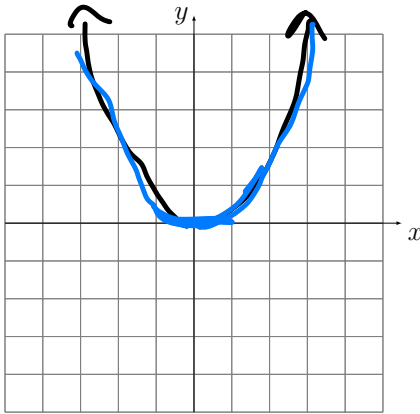
Linear Functions $f(x) = mx + b$; $m = \text{slope}$, $b = \text{y-intercept}$



Power Functions - Positive Exponents

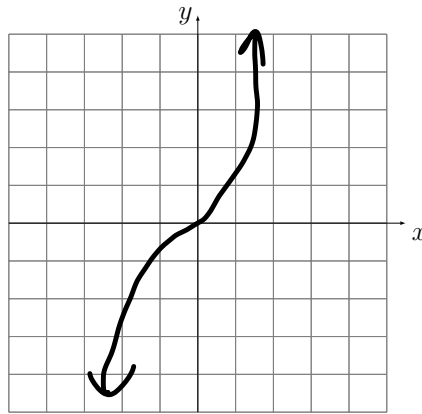
$$f(x) = K x^p$$

p - positive integer



$$f(x) = x^2$$

$$f(x) = x^4$$

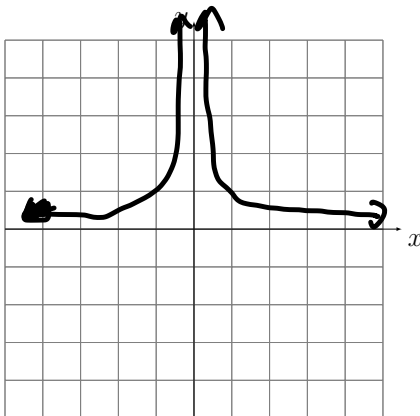


$$f(x) = x^3$$

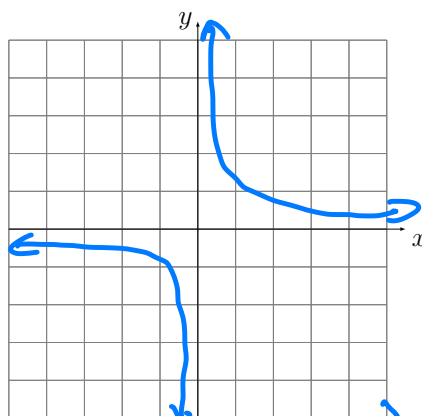
Power Functions - Negative Exponents

$$f(x) = K x^p$$

p - negative integer

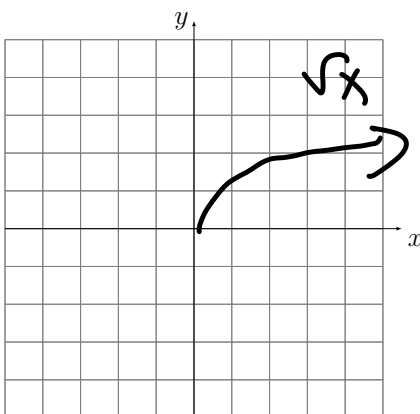


$$f(x) = x^{-2} = \frac{1}{x^2}$$

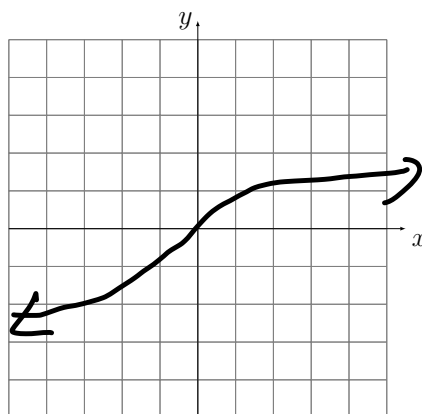


$$f(x) = x^{-3} = \frac{1}{x^3}$$

Root Functions



$$f(x) = \sqrt{x}$$



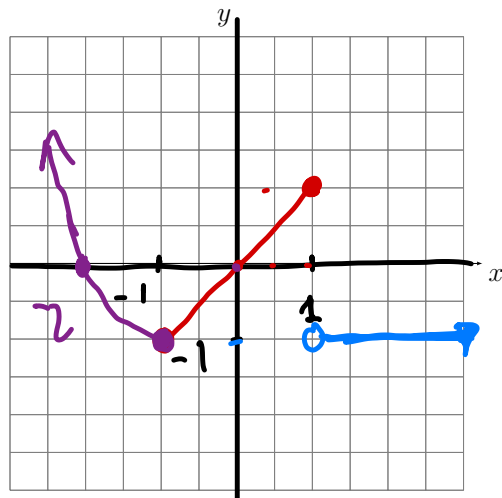
$$f(x) = \sqrt[3]{x}$$

2. Graphing Piecewise defined functions

Graph the function

$$f(x) = \begin{cases} x^2 + 2x & x \leq -1 \\ x & -1 < x \leq 1 \\ -1 & 1 < x \end{cases}$$

parabola
line
horizontal line



parabola $x^2 + 2x$
 $x : x^2 + 2x = x(x+2)$

vertex: $(-\frac{b}{2a}, y)$

up ✓

$$x^2 + 2x$$

$$ax^2 + bx + c$$

$$a = 1$$

$$b = 2$$

$$\text{vertex: } -\frac{b}{2a} = -\frac{2}{2(1)}$$

$$\begin{aligned} x \text{ coord} &= -1 \\ y \text{ coord} &= (-1)^2 + (2)(-1) \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

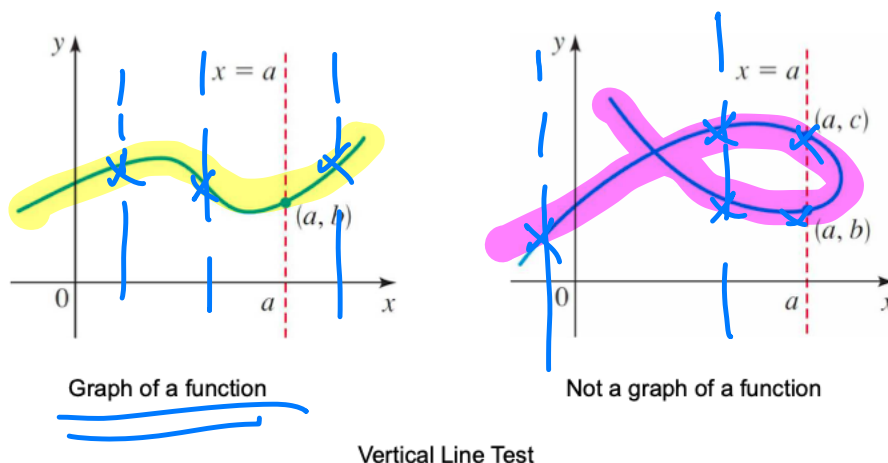
$$\text{vertex } (-1, -1)$$

Steps to graph a piecewise function:

1. Graph each piece on its given domain
2. Use solid dots (•) for included endpoints
3. Use open dots (○) for excluded endpoints
4. Check that the pieces connect as specified

3. Function or not a function?

Given a graph, one can decide if the graph represents a function if the graph passes the **vertical line test**: A **curve in the xy-plane** represents a function if and only if no vertical intersects the curve more than once.



Given the equation of the relationship between x and y , one can decide if y represents a function of x by solving for y and seeing if each x value has exactly one y value assigned to it.

Examples: Does the equation define y as a function of x ?

1. $y - x^2 = 2$

$y = 2 + x^2$ yes

2. $x^2 + y^2 = 4$

$y^2 = 4 - x^2$
 $\sqrt{y^2} = \pm \sqrt{4 - x^2}$
 $y = \pm \sqrt{4 - x^2}$

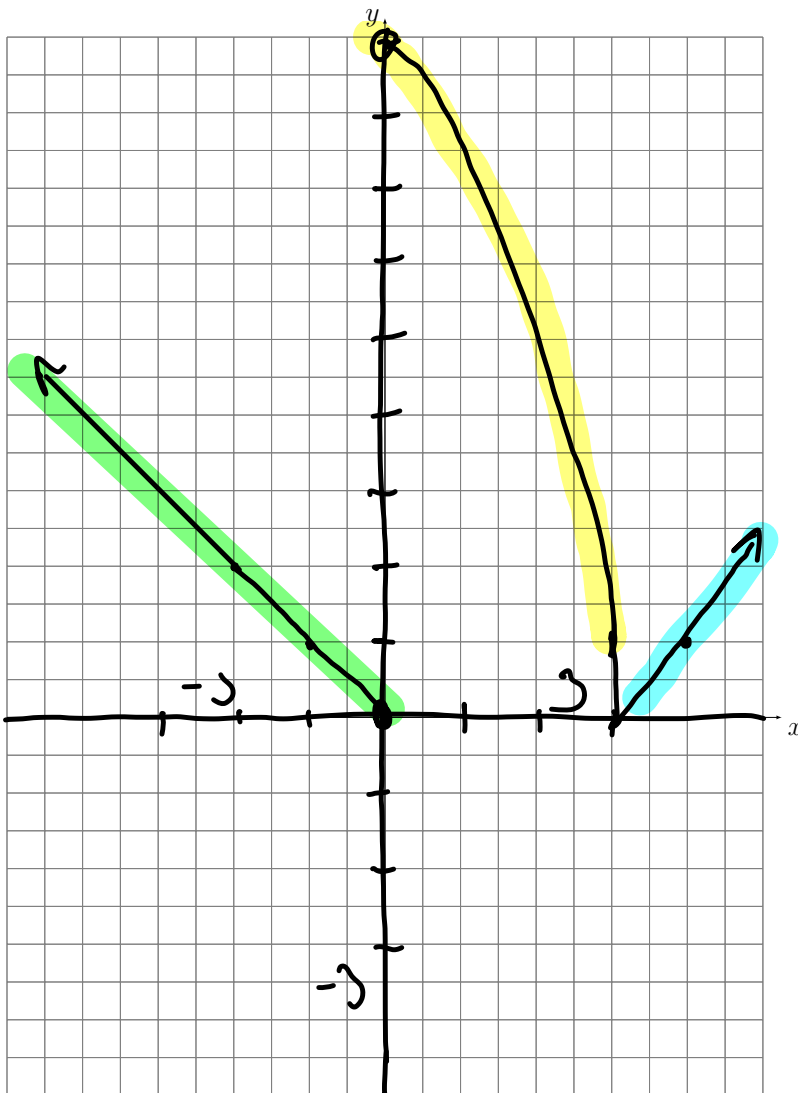
(square root both sides)

$x = 0$
 $y = \pm \sqrt{4 - 0^2} = \pm 2$
 No!

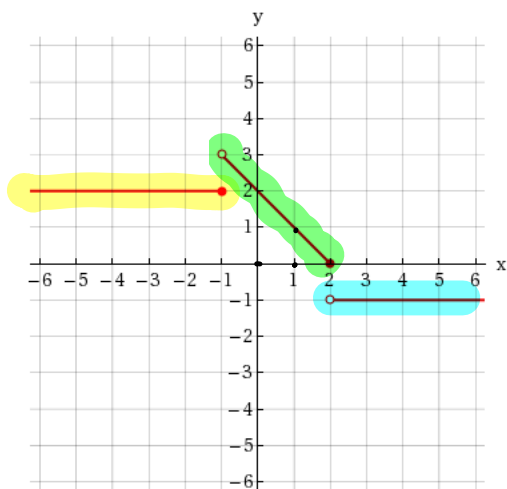
Practice: Graph the following function by plotting points.

$$f(x) = \begin{cases} -x & x \leq 0 \\ 9 - x^2 & 0 < x \leq 3 \\ x - 3 & 3 < x \end{cases}$$

line : $m = -1, b = 0$
parabola down
line : $m = 1, b = -3$



Practice: Find a formula for the function and state its domain and range.



$$f(x) = \begin{cases} 2, & x \leq -1 \\ -x+2, & -1 < x \leq 2 \\ -1, & x > 2 \end{cases}$$

Increasing/decreasing/constant function

Let f be a function.

- A function is **increasing** on an interval if as x increases, $f(x)$ increases.

$$\text{if } x_1 < x_2 \text{ then } f(x_1) < f(x_2)$$



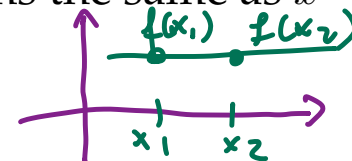
- A function is **decreasing** on an interval if as x increases, $f(x)$ decreases.

$$\text{if } x_1 < x_2 \text{ then } f(x_1) > f(x_2)$$

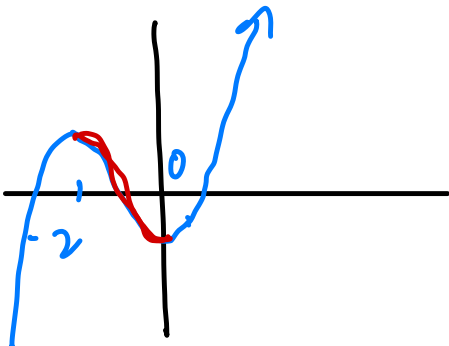


- A function is **constant** on an interval if $f(x)$ remains the same as x changes.

$$\text{if } x_1 < x_2 \text{ then } f(x_1) = f(x_2)$$



Example 1. Graph the function $f(x) = x^3 + 3x^2 - 1$. Then use the graph to describe the increasing and decreasing behavior of the function.

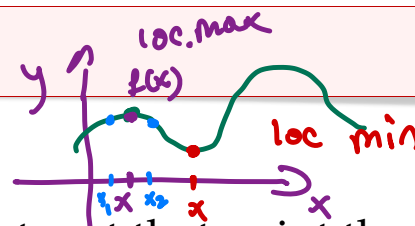


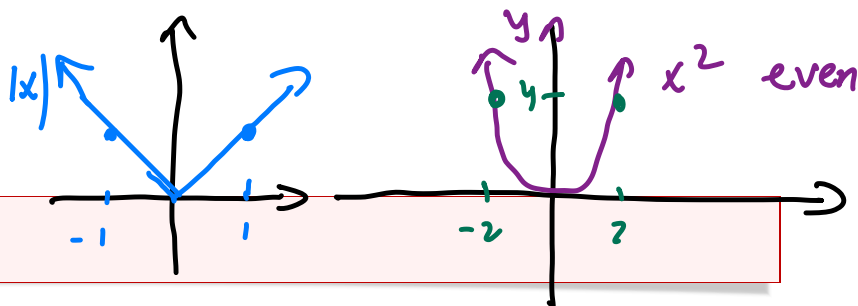
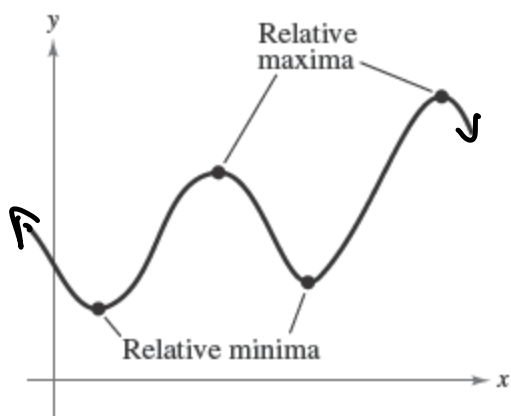
increasing: $(-\infty, -2) \cup (0, \infty)$
decreasing: $(-2, 0)$

Relative (local) maxima/minima

For a function f :

- A point is a **local maximum** if $f(x)$ is greater at that point than at nearby points.
- A point is a **local minimum** if $f(x)$ is less at that point than at nearby points





Even/odd functions

- A function is **even** if its graph is symmetric about the y-axis

$$f(1) = 1$$

$$f(-1) = 1$$

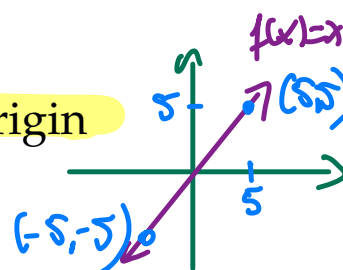
$$f(-x) = f(x) \text{ for all } x \text{ in the domain}$$

$$f(-1) = f(1)$$

- A function is **odd** if its graph is symmetric about the origin

$$f(-x) = -f(x) \text{ for all } x \text{ in the domain}$$

$$f(-5) = -f(5)$$



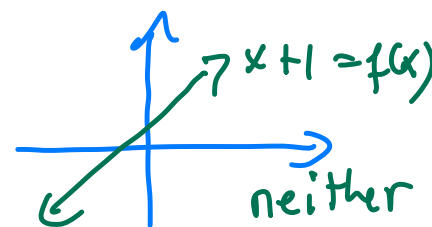
Example 2. Determine whether the function is even, odd or neither.

1. $f(x) = x^3 + 4x$

2. $g(x) = 3x - x^2$

3. $f(t) = 5t^{2/3}$

$$(-x)^3 = (-x)(-x)(x) = x^2(-x)$$



$$1) f(-x) = (-x)^3 + 4(-x) = -x^3 - 4x = -(x^3 + 4x) = -f(x)$$

odd

$$2) g(-x) = 3(-x) - (-x)^2 = -3x - x^2$$

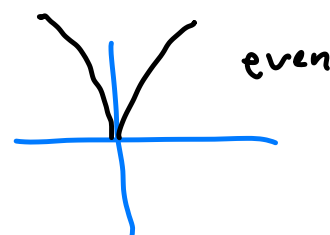
neither

this is not $g(x)$ or $-g(x)$

$$3x - x^2 \quad -3x + x^2$$

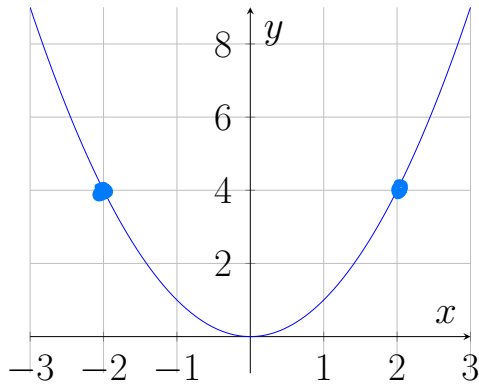
$$3) f(-t) = 5(-t)^{2/3} = 5\sqrt[3]{(-t)^2} = 5\sqrt[3]{t^2} = 5t^{2/3} = f(t)$$

even



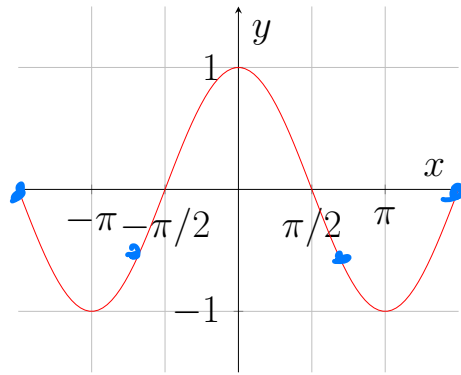
even

Function: $f_1(x) = x^2$



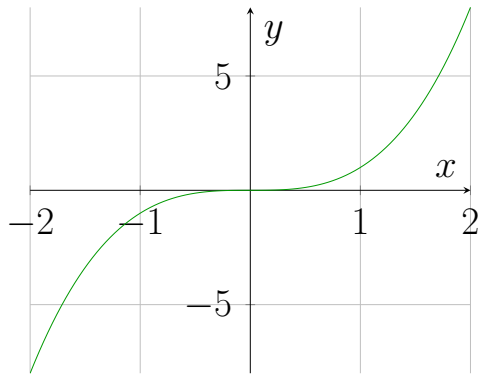
Function: $f_2(x) = \cos(x)$

even



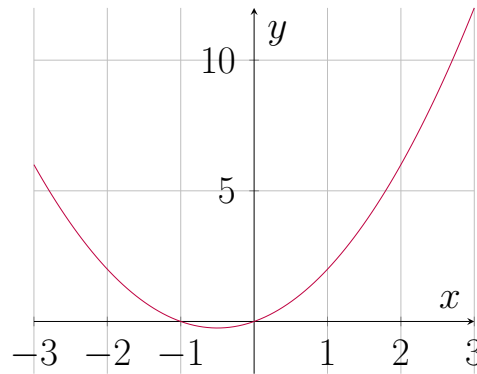
odd

Function: $f_3(x) = x^3$



$f_4(x) = x^2 + x$

neither



Properties

- For even functions (f_1 and f_2), $f(x) = f(-x)$ for all x in the domain
- For the odd function (f_3), $f(-x) = -f(x)$ for all x in the domain
- For f_4 , neither property holds: $f_4(-x) \neq f_4(x)$ and $f_4(-x) \neq -f_4(x)$